Dust dynamics in disks

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Outline

1. **Background on protoplanetary disks**
2. Dust dynamics

Protoplanetary disk structure

- Equilibrium structure of gas orbiting a star
  - steady-state solution to
    - hydrodynamic equations (continuity, Navier-Stokes)
    - gravity equation (Poisson)

  ⇒ non trivial, stability not guaranteed

- Protoplanetary disks: simplifications
  - $M_{\rm disk} \ll M_\star$ ⇒ neglect disk gravitational potential
    - valid for most observed disks ($M_{\rm disk}$ ~ 0.01 $M_\odot$)
    - marginal for some ($M_{\rm disk}$ ~ 0.1 $M_\odot$)
  - geometrically thin: $H \ll r$, cylindrical symmetry
  - vertical and radial structures decoupled

Vertical hydrostatic equilibrium

- Euler equation for stationary (and inviscid) disk
  - $\frac{1}{\rho}\frac{\partial P}{\partial z} = g_z \simeq -\Omega_{\mathrm{K}}^2 z$
- Assumption of vertically isothermal disk
  - scale height: $H = \frac{c_{\mathrm{s}}}{\Omega_{\mathrm{K}}}$
  - $\frac{1}{\rho}\frac{\partial \rho}{\partial z} = -\frac{z}{H^2} \;\Rightarrow\; \rho(r,z) = \frac{\Sigma(r)}{\sqrt{2\pi}H(r)}\mathrm{e}^{-z^2/2H^2}$
  - $\frac{H}{r} = \frac{c_{\mathrm{s}}}{v_{\mathrm{K}}} = \frac{1}{\mathcal{M}}$: thin disk ⇔ supersonic
- Verification of hypotheses
  - $M_\star = 1\ M_\odot,\ r = 1\ \mathrm{AU}, T = 300\ \mathrm{K}$
    $\Rightarrow c_{\mathrm{s}} = 1.03\ \mathrm{km\,s^{-1}} \ll v_{\mathrm{K}} = 29.8\ \mathrm{km\,s^{-1}},\ \frac{H}{r} \simeq 3\%$
  - $g_{z,\star} > g_{z,\mathrm{disk}} \;\Leftrightarrow\; \frac{M_{\mathrm{disk}}}{M_\star} < \frac{1}{2}\frac{H}{r}$: OK for most disks

Power-law disks

- $\Sigma(r) = \Sigma_0 \left(\frac{r}{r_0}\right)^{-p} \;\Rightarrow\; M_\mathrm{disk} = \int_{r_\mathrm{in}}^{r_\mathrm{out}} \Sigma(r)\, 2\pi r \,\mathrm{d}r$
- $T(r) = T_0 \left(\frac{r}{r_0}\right)^{-q} \;\Rightarrow\; c_\mathrm{s} = c_\mathrm{s0} \left(\frac{r}{r_0}\right)^{-\frac{q}{2}}, \; c_\mathrm{s0} = \sqrt{\frac{k_\mathrm{B} T_0}{\mu m_\mathrm{H}}}$
- $v_\mathrm{K} = \sqrt{\frac{G M_\star}{r_0}} \left(\frac{r}{r_0}\right)^{-\frac{1}{2}}, \; \Omega_\mathrm{K} = \sqrt{\frac{G M_\star}{r_0^3}} \left(\frac{r}{r_0}\right)^{-\frac{3}{2}}$
- $H = \frac{c_\mathrm{s0}}{\Omega_\mathrm{K0}} \left(\frac{r}{r_0}\right)^{\frac{3-q}{2}}, \; \frac{H}{r} = \frac{c_\mathrm{s0}}{v_\mathrm{K0}} \left(\frac{r}{r_0}\right)^{\frac{1-q}{2}}$
    - $\frac{\partial}{\partial r}\left(\frac{H}{r}\right) > 0 \;\Leftrightarrow\; q < 1$: flared disk
      star visible from whole surface
- $\rho(r,z) = \frac{\Sigma_0}{\sqrt{2\pi} H_0} \left(\frac{r}{r_0}\right)^{-\left(p - \frac{q}{2} + \frac{3}{2}\right)} \mathrm{e}^{-z^2/2H^2}$
- $P(r,z) = \frac{c_\mathrm{s0}^2 \Sigma_0}{\sqrt{2\pi} H_0} \left(\frac{r}{r_0}\right)^{-\left(p + \frac{q}{2} + \frac{3}{2}\right)} \mathrm{e}^{-z^2/2H^2}$

Azimuthal motion

- Axisymmetric stationary flow with star's gravity only
- Radial component of Euler equation
  - $\dfrac{v_\theta^2}{v_\mathrm{K}^2} = 1 - 2\eta < 1$
  - sub-Keplerian parameter: $\eta = \dfrac{1}{2}\left(\dfrac{H}{r}\right)^2 \left(-\dfrac{\partial \ln P}{\partial \ln r}\right) > 0$
- Power-law disk: $\eta = \dfrac{1}{2}\left(p + \dfrac{q}{2} + \dfrac{3}{2}\right)\left(\dfrac{H}{r}\right)^2 \ll 1$
  - ex: $\dfrac{H}{r} = \mathrm{cst} = 0.05 \ (\Rightarrow q = 1), \ p = 1$
    - $\Rightarrow \eta = 3.75 \times 10^{-3} \Rightarrow v_\theta \simeq 0.996 v_\mathrm{K}$
- Gas only: approximation $v_\theta = v_\mathrm{K}$ OK
- Gas+dust: deviation crucial for evolution of solids

Protoplanetary disk evolution

- Disks are not static, they evolve slowly
  - theoretical explanation not easy
- Specific angular momentum in geometrically thin disk
  - $j = rv_\theta = \sqrt{GM_\star r}$: *increasing* function of *r*

    ⇒ accreting gas needs to lose angular momentum

- Angular momentum transport
  - central problem in all accretion disks
    - redistribution within disk (due to "viscosity")?
    - loss from system (magnetic wind)?
  - difficult because subtle effect
  - PPD lifetime: several Myr ~10^4 dynamical times at r_{out}

    ⇒ almost, but not quite, stable

Surface density evolution

- If static potential, local J conservation ⇒ Σ constant
- Accretion and disk evolution: redistribution of J
  - gas loses J and spirals towards the star
  - elsewhere, gas gains J and move outwards
- Evolution of $\Sigma(r,t)$
  - continuity equation: $r\dfrac{\partial \Sigma}{\partial t} + \dfrac{\partial}{\partial r}(r\Sigma v_r) = 0$
  - azimuthal component of momentum equation
    - $r\dfrac{\partial}{\partial t}(r^2\Omega\Sigma) + \dfrac{\partial}{\partial r}(r^2\Omega . r\Sigma v_r) = \dfrac{1}{2\pi}\dfrac{\partial \Gamma}{\partial r}$
      - viscous torque $\Gamma = 2\pi r . \nu \Sigma r \dfrac{\mathrm{d}\Omega}{\mathrm{d}r} . r, \;\; \nu :$ viscosity

$$\Rightarrow \frac{\partial \Sigma}{\partial t} = \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}(\nu\Sigma r^{1/2})\right]$$
: diffusion equation

Viscous time scale

- Change of variable
  - $X = 2r^{1/2}$
  - $f = \frac{3}{2}\Sigma X$

  $\Rightarrow$ diffusion equation: $\frac{\partial f}{\partial t} = D\frac{\partial^2 f}{\partial X^2}$

    - diffusion coefficient $D = \frac{12\nu}{X^2}$

  - characteristic diffusion timescale: X^2/D

  $\Rightarrow$ viscous timescale $\tau_\nu \simeq \frac{r^2}{\nu}$

    - can be estimated observationally
      - e.g. decay rate of accretion

    $\Rightarrow$ estimate of effective viscosity

Time-dependent solution

- ν = cst, solution: Green's function
  - t = 0: ring of mass m at r_0
    - $\Sigma(r, t=0) = \dfrac{m}{2\pi r_0}\delta(r - r_0)$
  - zero-torque BC at r = 0

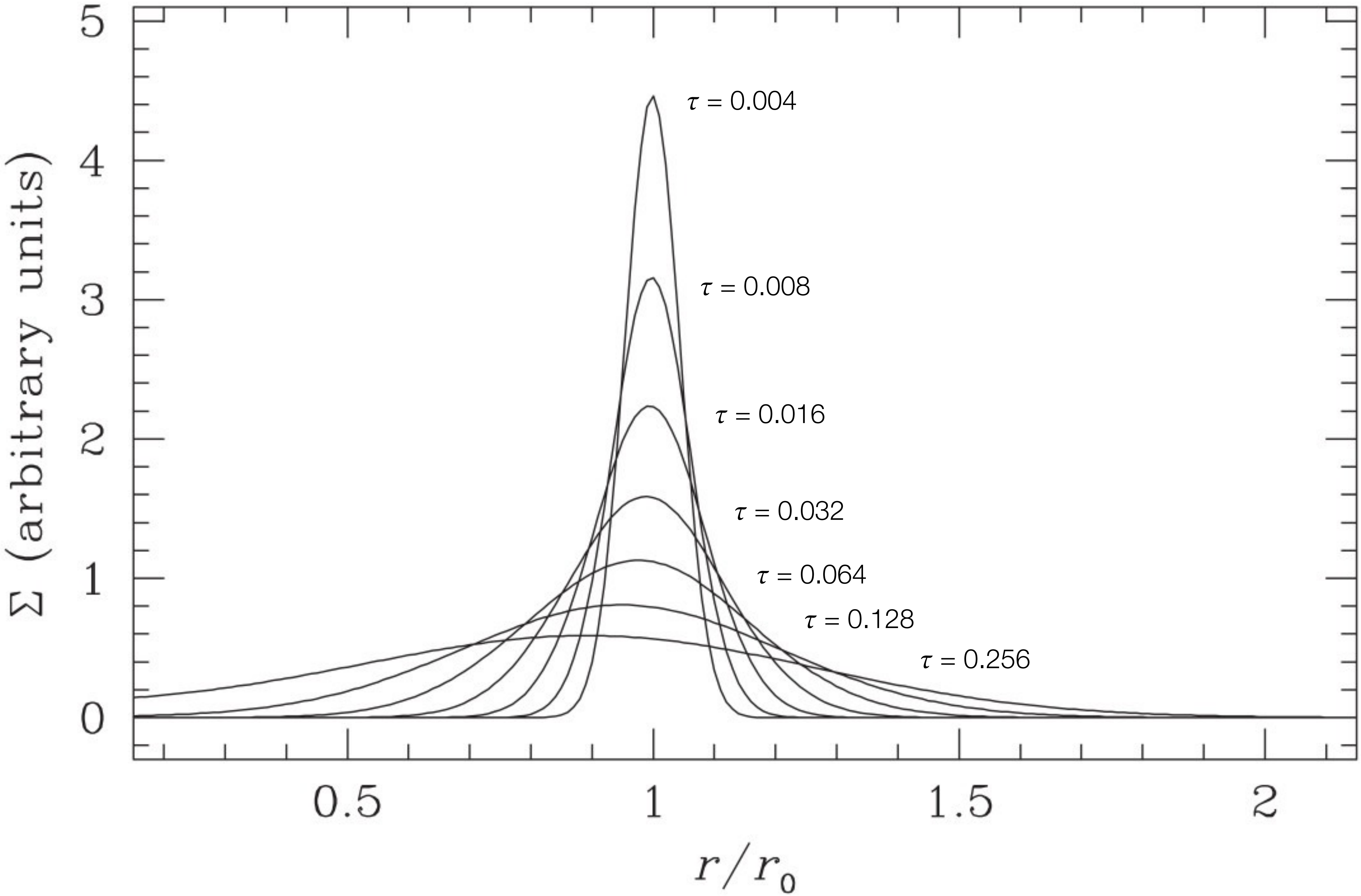
$$\Rightarrow \Sigma(x,\tau) = \frac{m}{\pi r_0^2}\frac{1}{\tau}x^{-1/4}\mathrm{e}^{-(1+x^2)/\tau}I_{1/4}\left(\frac{2x}{\tau}\right)$$

    - $x = r/r_0, \;\; \tau = 12\nu r_0^{-2}t$
    - $I_{1/4}$: modified Bessel function of the first kind
  - ring spreading
    - mass flows towards r = 0
    - angular momentum carried towards r = +∞ by negligible fraction of mass

Lynden-Bell & Pringle 1974

Time-dependent solution

Solution for ν = cst



Time-dependent solution

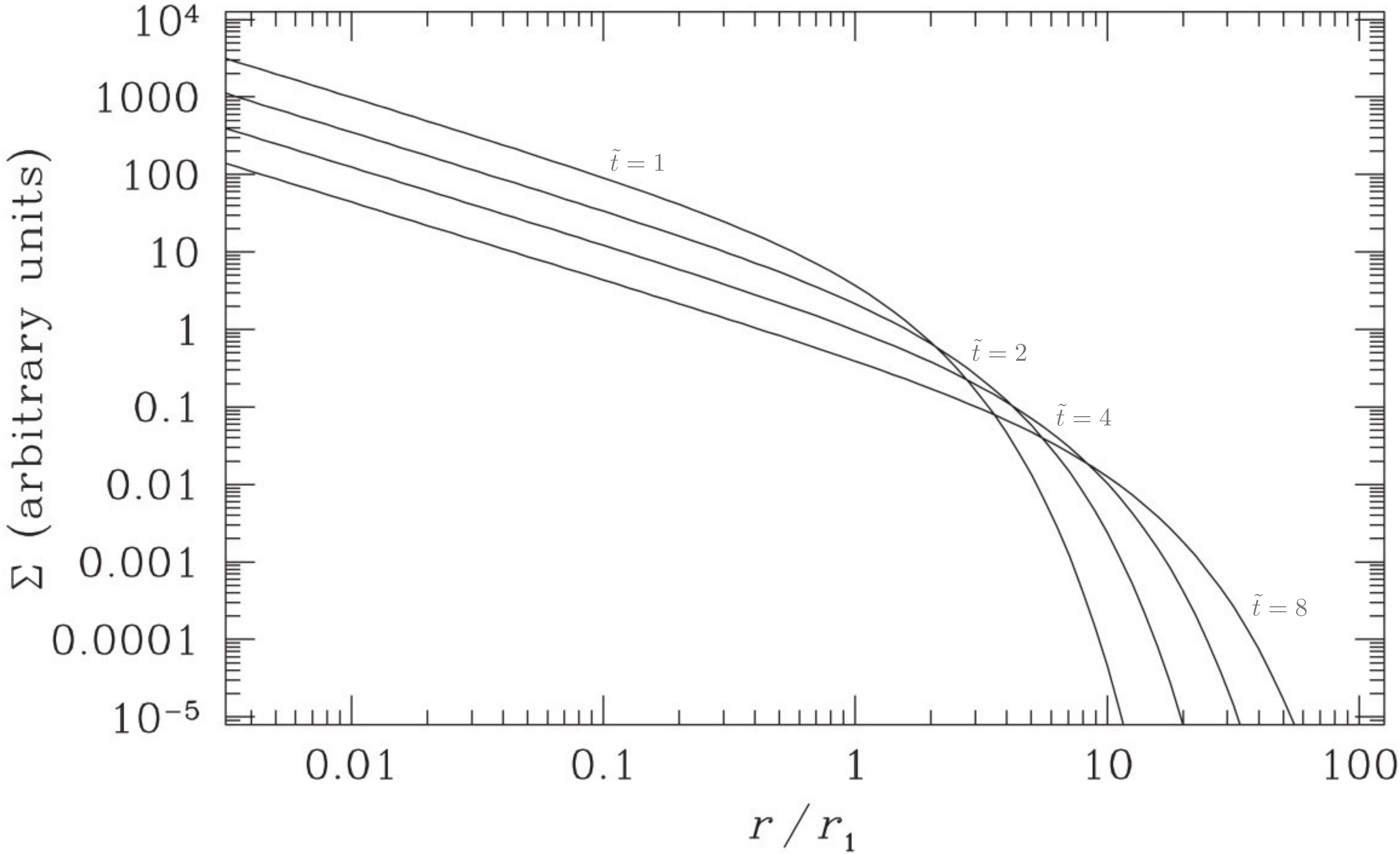
- $\nu \propto r^\gamma$
  - t = 0: stationary solution → r_1 w/ exponential cutoff
    - $\Sigma(t=0) = \dfrac{C}{3\pi\nu_1\tilde{r}^\gamma}\mathrm{e}^{-\tilde{r}^{2-\gamma}}$
    - $\tilde{r} = r/r_1,\ \nu_1 = \nu(r_1),\ C:$ normalization cst

  ⇒ solution: $\Sigma(\tilde{r},\tilde{t}) = \dfrac{C}{3\pi\nu_1\tilde{r}^\gamma}\tilde{t}^{-\frac{5/2-\gamma}{2-\gamma}}\mathrm{e}^{-\tilde{r}^{2-\gamma}/\tilde{t}}$
    - $\tilde{t} = \dfrac{t}{t_\mathrm{s}} + 1,\ t_\mathrm{s} = \dfrac{1}{3(2-\gamma)^2}\dfrac{r_1^2}{\nu_1}$
    - allows to compute M_disk and $\dot{M}$
  - disk mass decreases
  - characteristic scale increases

Lynden-Bell & Pringle 1974

Time-dependent solution

Self-similar solution for $\gamma = 1$



Angular momentum transport

- Physical origin
  - molecular viscosity: $\nu_\mathrm{m} = \lambda c_\mathrm{s}$
    - mean free path: $\lambda = \dfrac{1}{n\sigma_\mathrm{mol}}$
    - cross-section for molecular collisions: σ_mol
      - H_2 gas: $\sigma_\mathrm{mol} \simeq 2 \times 10^{-19}\ \mathrm{m}^2$
    - $r = 10\,\mathrm{AU},\ c_\mathrm{s} = 0.5\,\mathrm{km\,s^{-1}}, n = 10^{18}\,\mathrm{m}^{-3}$

  $\Rightarrow \nu_\mathrm{m} = 2500\,\mathrm{m^2\,s^{-1}} \ \Rightarrow\ \tau_{\nu_\mathrm{m}} = r^2/\nu_\mathrm{m} \simeq 3 \times 10^{13}\ \mathrm{yr}$!!!

  ⇒ molecular viscosity NOT the source of *J* transport

  - Reynolds number for $U = c_\mathrm{s},\ L = H = 0.05r,\ r = 10\ \mathrm{AU}$
    - $\mathrm{Re} = UL/\nu_\mathrm{m} \sim 10^{10}$

  ⇒ if instabilities present, disk can be highly turbulent

Shakura-Sunyaev α prescription

- Turbulent disk
  - macroscopic mixing ⇒ "effective" or "turbulent" viscosity
  - if isotropic turbulence
    - turbulent flow outer scale < H: smallest scale in disk
    - turbulent velocity < c_s:
      - supersonic motion ⇒ shocks and dissipation

  ⇒ "turbulent" viscosity: $\nu = \alpha c_{\rm s} H$
  - $\alpha < 1$: dimensionless Shakura-Sunyaev parameter
  - no reason for α to be constant
    - often taken constant…

Shakura & Sunyaev 1973

Magneto-Rotational Instability (MRI)

- Linear stability criterion of a weakly magnetized disk flow
  - $\frac{\mathrm{d}}{\mathrm{d}r}(\Omega^2) > 0$ *Velikhov 1959; Chandrasekhar 1961; Balbus & Hawley 1991*
  - *not* satisfied in Keplerian disk

  ⇒ linear instability known as MRI (or Balbus-Hawley inst.)

- Ideal MHD equations
  - continuity: $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$
  - momentum conservation:

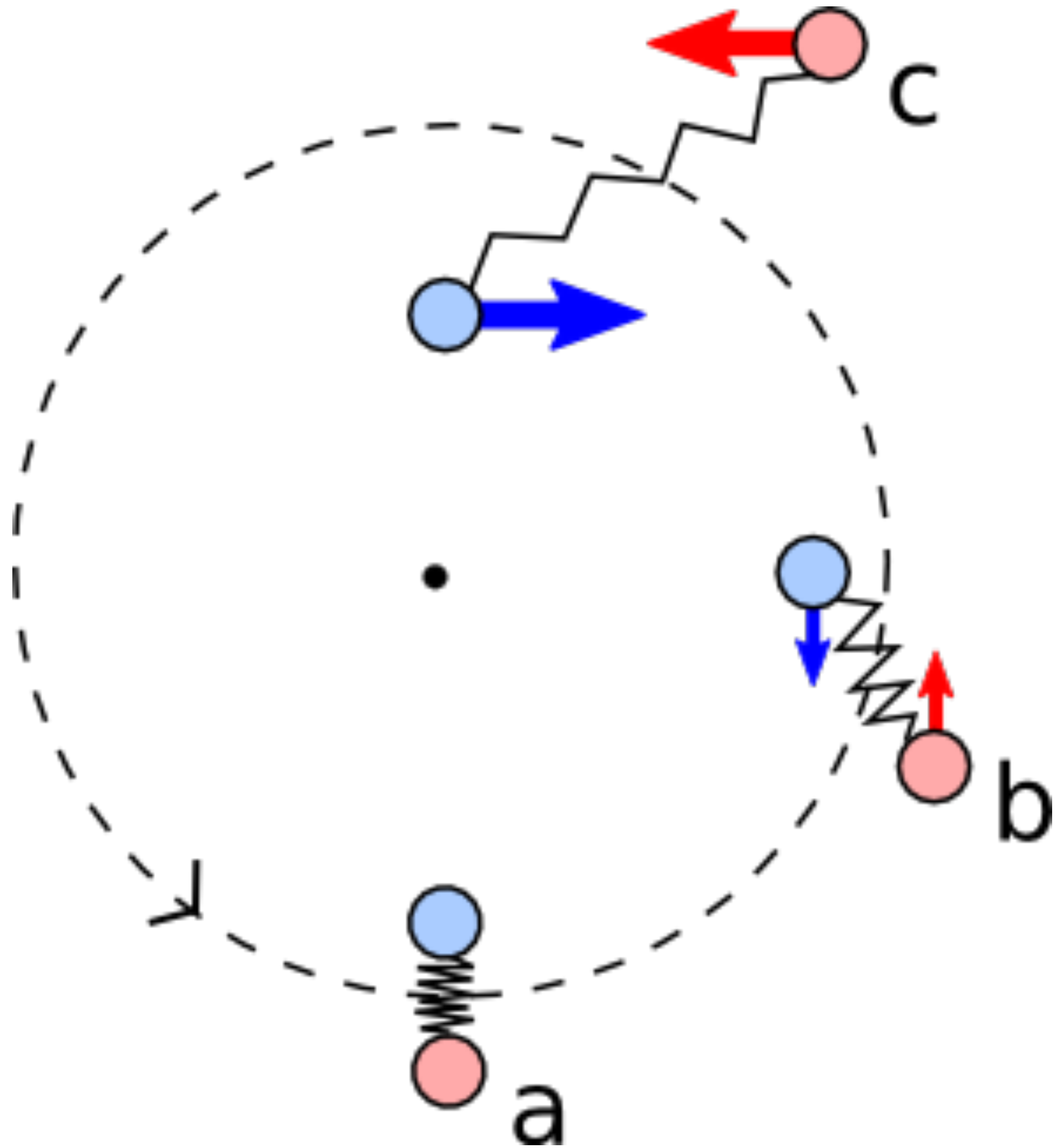
$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla}\left(P + \frac{B^2}{2\mu_0}\right) - \vec{\nabla}\Phi + \frac{1}{\mu_0 \rho}(\vec{B} \cdot \vec{\nabla})\vec{B}$$

magnetic pressure — magnetic tension

  - magnetic induction: $\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times (\vec{v} \times \vec{B})$

Magneto-Rotational Instability (MRI)

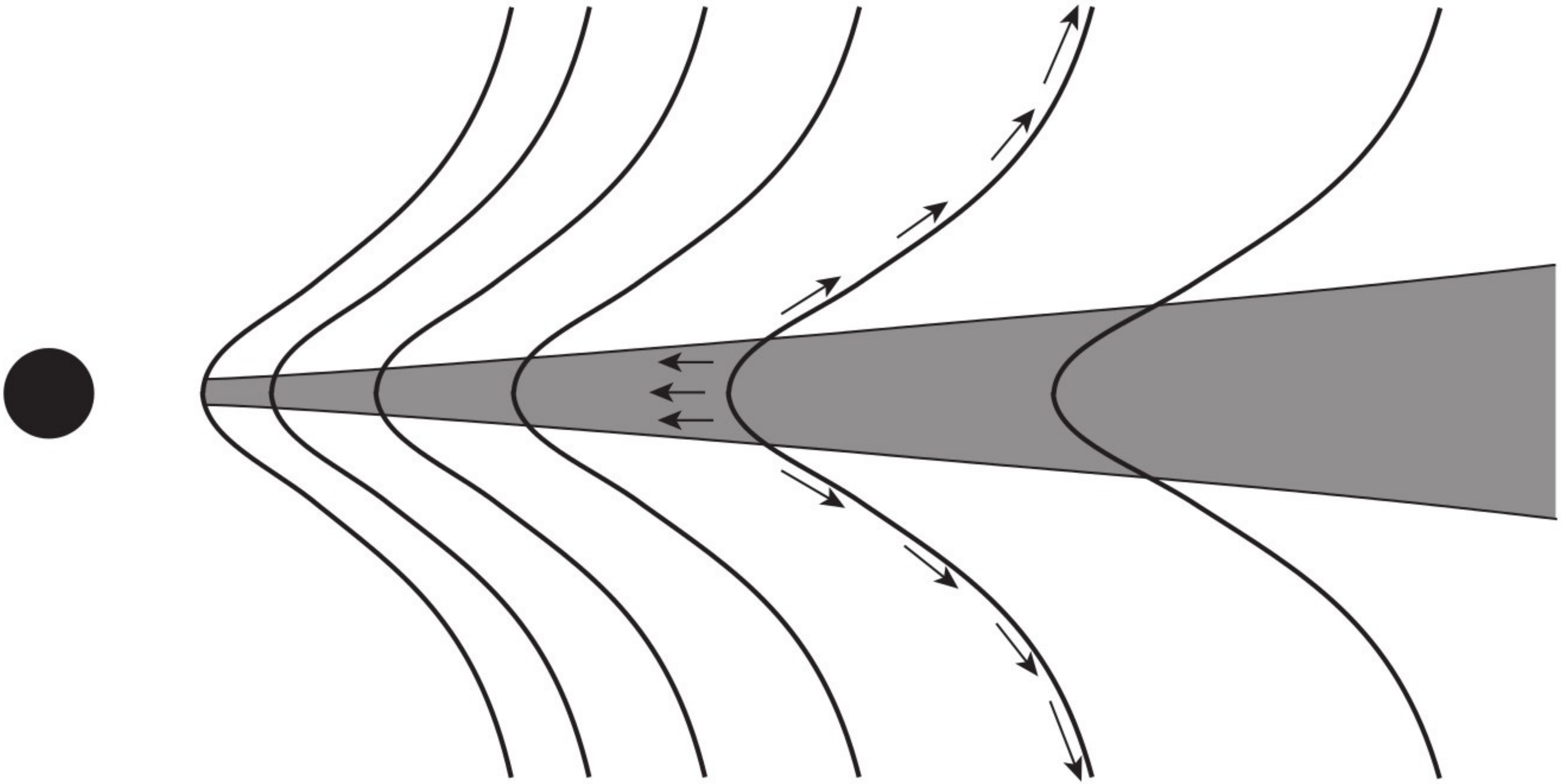
Physical origin of the MRI



Balbus+Hawley1991

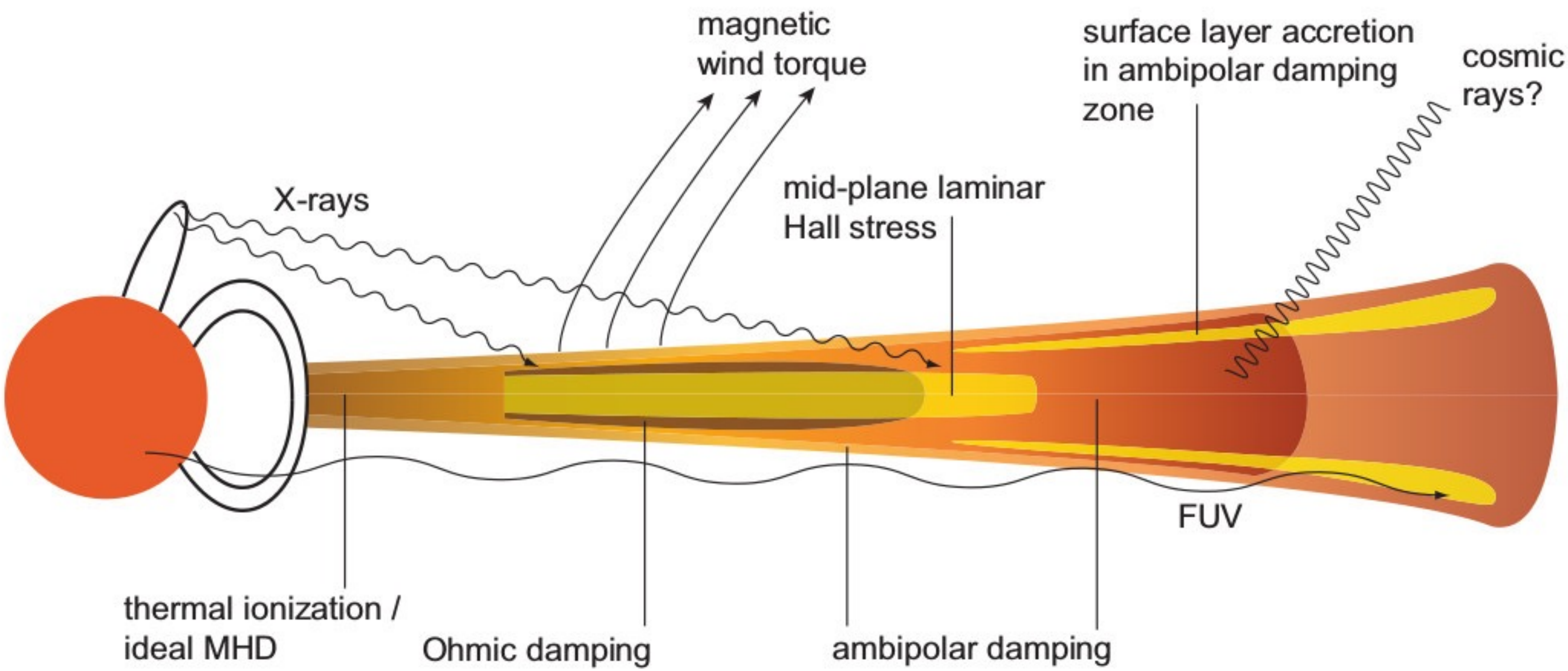
Disk winds and magnetic braking

Angular momentum loss instead of redistribution



Armitage 2010

Layered disks



Armitage 2014

Observed disk structures

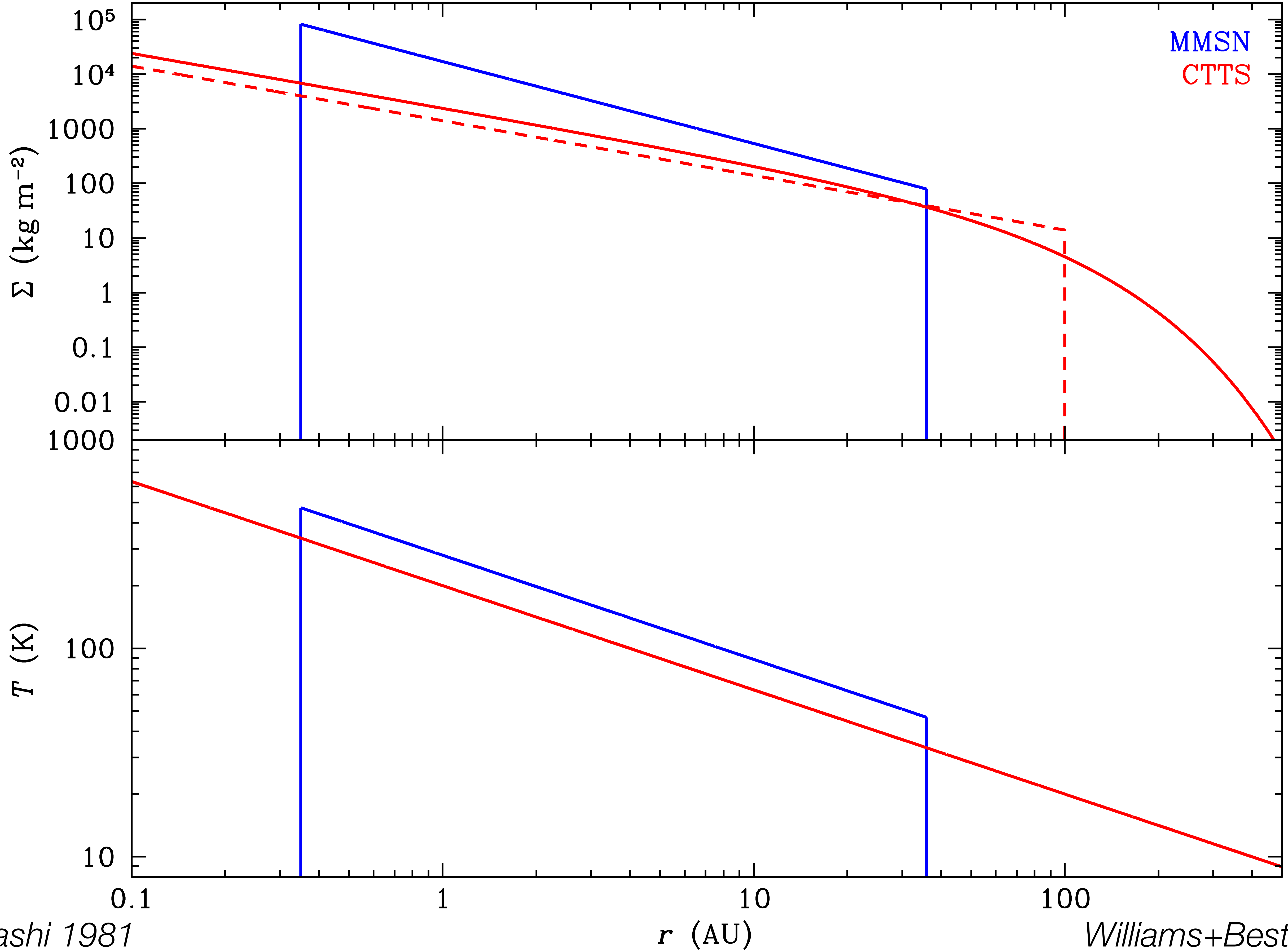
- MMSN
  - $r_{\rm in} = 0.35\ {\rm AU},\ r_{\rm out} = 36\ {\rm AU}$
  - $\Sigma_{\rm disk} = 17\,000 \left(\frac{r}{\rm AU}\right)^{-3/2}\ {\rm kg\,m^{-2}} \Rightarrow M_{\rm disk} = 0.013\ M_\odot$
  - $T_{\rm c} = 280 \left(\frac{r}{\rm AU}\right)^{-1/2}\ {\rm K}$

Hayashi1981

- Observed CTTS disks ⇒ median disk
  - $(r_{\rm in} = 0.1\ {\rm AU}),\ M_{\rm disk} = 0.01\ M_\odot$
  - $r_{\rm out} = 100\ {\rm AU} \Rightarrow \Sigma_{\rm disk} = 1\,400 \left(\frac{r}{\rm AU}\right)^{-1}\ {\rm kg\,m^{-2}}$
  - $r_{\rm c} = 60\ {\rm AU} \Rightarrow \Sigma_{\rm disk} = 40 \left(\frac{r}{60\ {\rm AU}}\right)^{-1} {\rm e}^{-r/60\ {\rm AU}}\ {\rm kg\,m^{-2}}$
  - $T_{\rm c} = 200 \left(\frac{r}{\rm AU}\right)^{-1/2}\ {\rm K}$

Williams+Best2014

Observed disk structures



Hayashi 1981

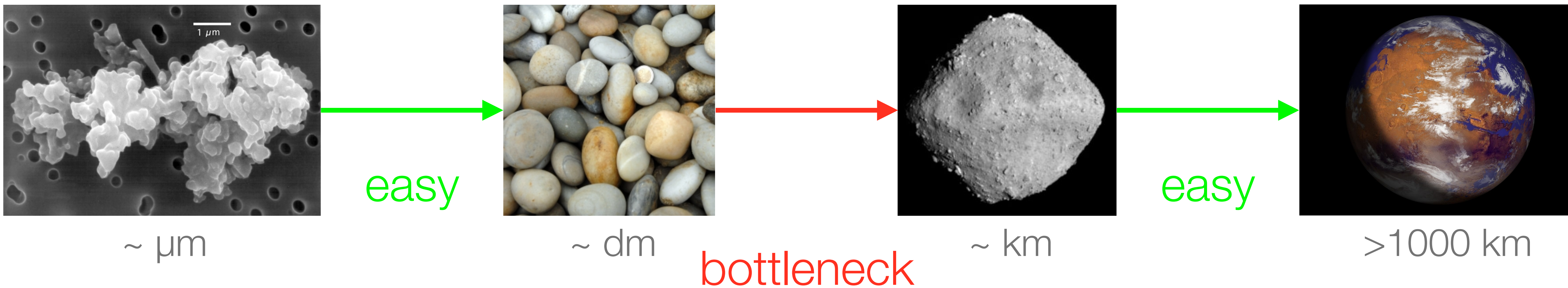
Williams+Best2014

Outline

1. Background on protoplanetary disks
2. **Dust dynamics**

Planet formation

- Core accretion paradigm



- Barriers of planet formation
  - Radial drift
    Adachi+1976, Weidenschilling1977, Nakagawa+1986, Birnstiel+2010, Laibe+2012,2014
  - Fragmentation
    Dullemond+Dominik2005, Blum+Wurm2008
  - Bouncing
    Zsom+2010, Windmark+2012

Dynamics of solid bodies in disks

- Evolution of "small" solids
  - dominated by aerodynamic forces b/w gas and solids
  - dust grains: sub-μm to mm in size
  - pebbles : 1—10 cm
  - rocks : ~1 m
- Planetesimals: $\gtrsim$ 1 km in radius
  - large enough for evolution dominated by gravitation

Aerodynamic drag force

- Solid spherical particle of radius *s*
- Relative velocity with the gas $\Delta\vec{v} = \vec{v}_\mathrm{d} - \vec{v}_\mathrm{g}$

$\Rightarrow$ drag force $\vec{F}_\mathrm{D} = -\frac{1}{2} C_\mathrm{D}\, \pi s^2\, \rho_\mathrm{g} \Delta v \Delta\vec{v}$

drag coefficient (C_D), cross section (πs^2), ram pressure ($\rho_\mathrm{g} \Delta v \Delta\vec{v}$)

- Form of C_D depends on size *s* vs. gas mean free path λ
  - *s* < 9/4 λ: Epstein regime, $C_\mathrm{D} = \frac{8 v_\mathrm{th}}{3\Delta v}$
    - momentum transfer by collisions with gas molecules
  - *s* > 9/4 λ: Stokes regime
    - gas flows as a fluid with $\mathrm{Re} = \frac{2s\Delta v}{\nu} = \frac{4s\Delta v}{v_\mathrm{th}\lambda}$
    - $\begin{cases} \mathrm{Re} < 1 & : C_\mathrm{D} = 24\mathrm{Re}^{-1} \\ 1 < \mathrm{Re} < 800 & : C_\mathrm{D} = 24\mathrm{Re}^{-0.6} \\ \mathrm{Re} > 800 & : C_\mathrm{D} = 0.44 \end{cases}$

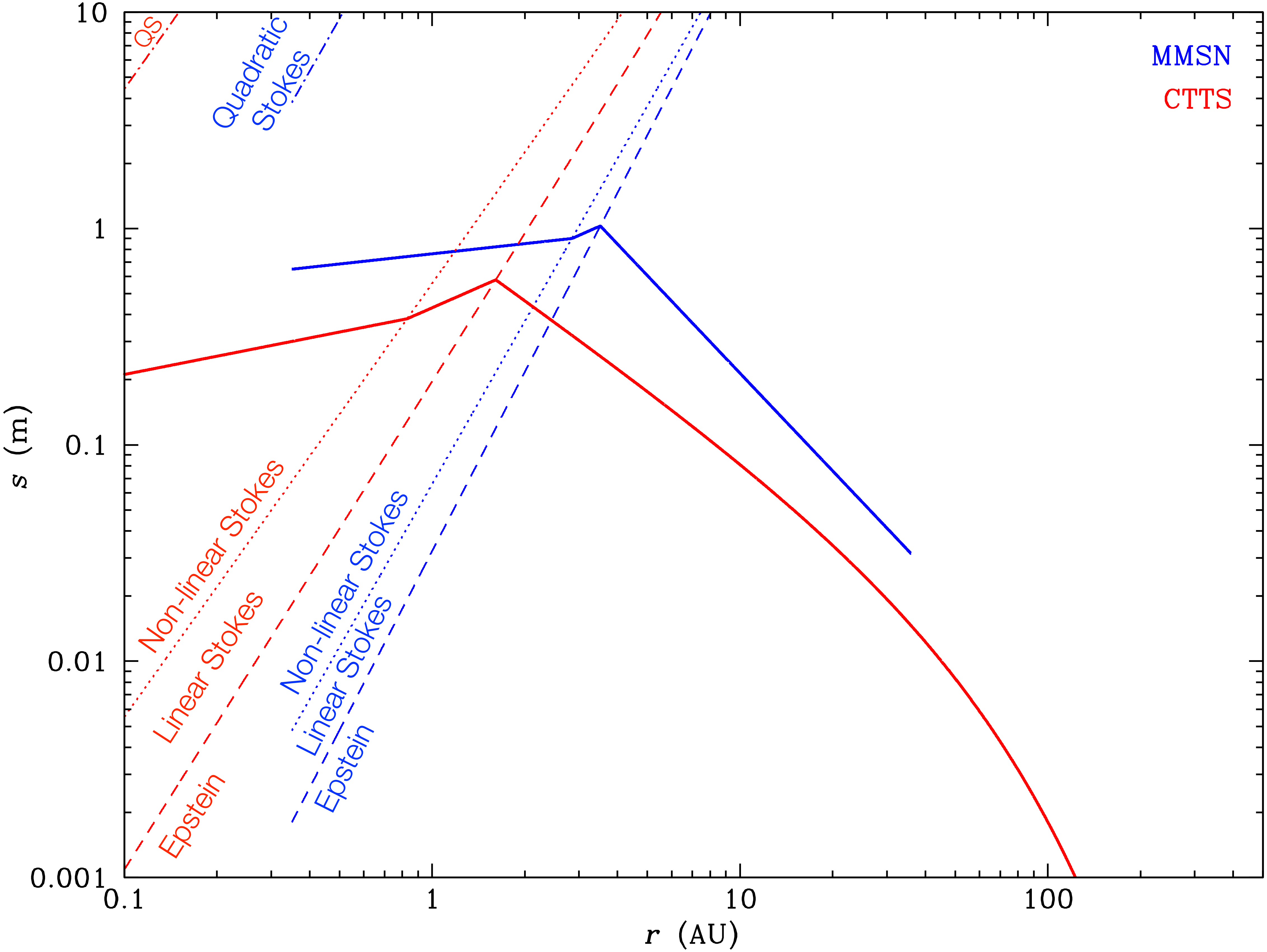
Whipple1972, Weidenschilling1977

Aerodynamic drag

- Stopping time $t_\mathrm{s} = \dfrac{m_\mathrm{d}\Delta v}{F_\mathrm{D}} = \dfrac{8\rho_\mathrm{s} s}{3C_\mathrm{D}\rho_\mathrm{g}\Delta v}$
  - mid-plane at 1 AU: t_s~1 s for s=1 µm
- Stokes number $\mathrm{St} = \dfrac{t_\mathrm{s}}{t_\mathrm{K}} = \Omega_\mathrm{K} t_\mathrm{s}$
  - St«1: small grains, strong coupling
  - St»1: large particles, weak coupling
  - **St~1**: largest effect of drag
  - Epstein regime: $\mathrm{St}_\mathrm{Ep} = \dfrac{\Omega_\mathrm{K}\rho_\mathrm{s} s}{\rho_\mathrm{g} v_\mathrm{th}} = \dfrac{\pi\rho_\mathrm{s} s}{2\Sigma_\mathrm{g}} \mathrm{e}^{z^2/2H^2}$

Aerodynamic drag

Grain size for which St=1 in the disk mid-plane, for ρ_S=1000 kg.m^{-3}

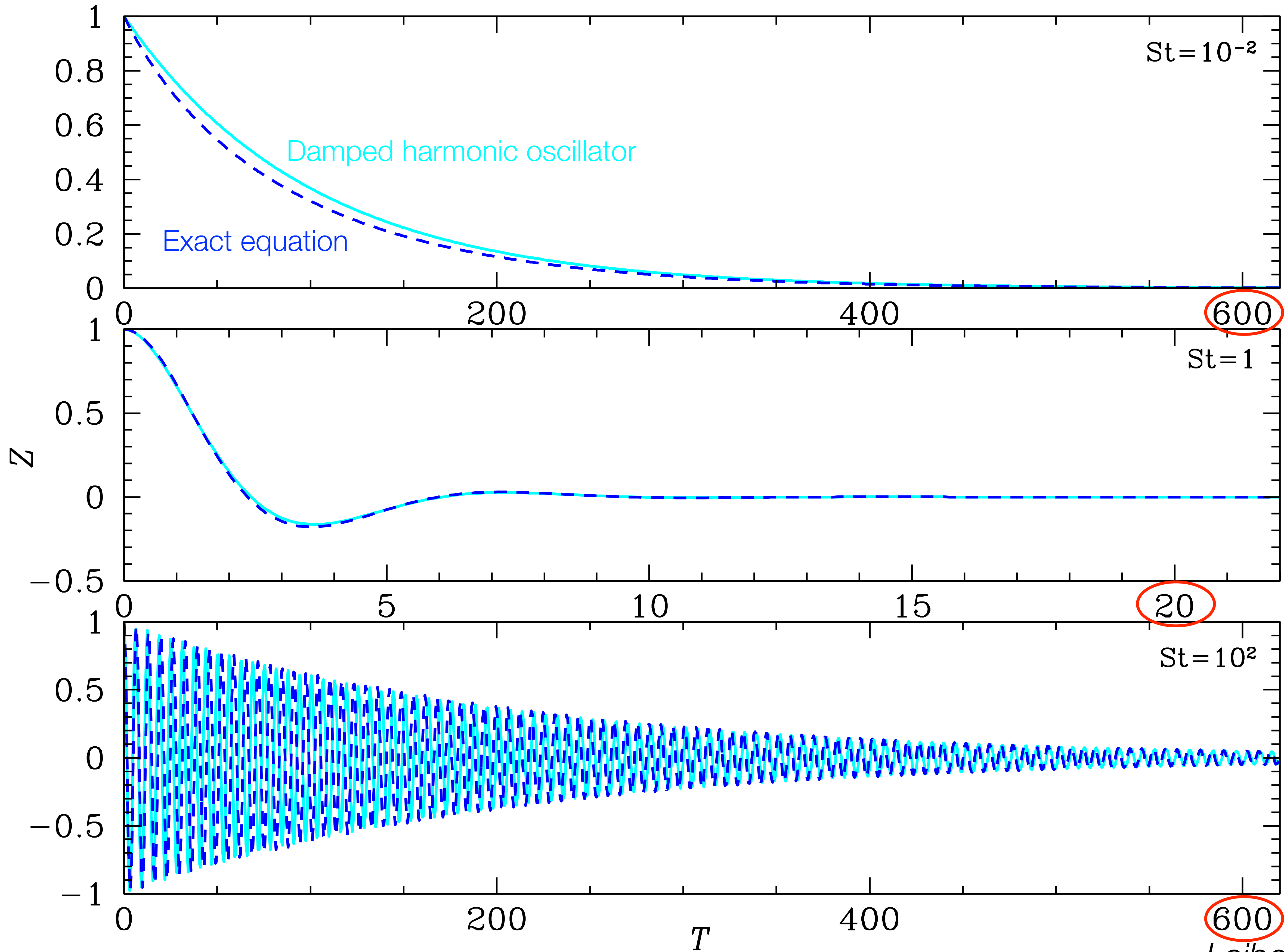


Vertical settling

- Vertical equation of motion of a particle in a laminar disk
  - $a_{\mathrm{d},z} = \dfrac{F_\mathrm{D}}{m_\mathrm{d}} - g_z = -\dfrac{\Delta v_z}{t_\mathrm{s}} - \Omega_\mathrm{K}^2 z$
  - no vertical gas motion: $\Delta v_z = v_{\mathrm{d},z}$
  - $Z = \dfrac{z}{z_0},\ T = \dfrac{t}{t_\mathrm{K}} \Rightarrow \ddot{Z} + \dfrac{\dot{Z}}{\mathrm{St}} + Z = 0$

  $\Rightarrow$ damped harmonic oscillator, settling time $T_\mathrm{set} \sim \dfrac{1+\mathrm{St}^2}{\mathrm{St}}$
  - small grains: terminal velocity quickly reached
    - $v_\mathrm{set} = \Omega_\mathrm{K}\mathrm{St}z,\ t_\mathrm{set} = \dfrac{z}{v_\mathrm{set}} = \dfrac{1}{\Omega_\mathrm{K}\mathrm{St}}$
    - Epstein: $v_\mathrm{set} = \dfrac{\Omega_\mathrm{K}^2 \rho_\mathrm{s} s}{\rho_\mathrm{g} c_\mathrm{s}} z$, $t_\mathrm{set} = \dfrac{\Sigma}{\sqrt{2\pi}\Omega_\mathrm{K}\rho_\mathrm{s} s} \mathrm{e}^{-z^2/2H^2}$
    - $s = 1\,\mu\mathrm{m},\ \rho_\mathrm{s} = 10^3\,\mathrm{kg\,m^{-3}},\ r = 1\ \mathrm{AU},\ z = H$:
      - $v_\mathrm{set} \sim 10^{-4}\ \mathrm{m\,s^{-1}},\ t_\mathrm{set} \sim 10^5\ \mathrm{yr} < t_\mathrm{disk}$

Vertical settling



Laibe+2014

Vertical settling with turbulence

- Turbulence stirs up dust, prevents efficient settling
- Small grains, turb. diffusion coeff. D_t=turb. visc. $\nu = \alpha c_\mathrm{s} H$
  - $t_\mathrm{stir} = \dfrac{z^2}{\nu} = t_\mathrm{set} \Rightarrow \dfrac{\sqrt{2\pi}\rho_\mathrm{s} s}{\alpha\Sigma} = \dfrac{\mathrm{e}^{-z^2/2H^2}}{z^2/H^2}$
  - stirring up to $z \gtrsim H \Rightarrow \alpha \gtrsim \dfrac{\sqrt{2\pi}\mathrm{e}^{1/2}\rho_\mathrm{s} s}{\Sigma} \sim 10^{-2}\left(\dfrac{s}{\mathrm{cm}}\right)$
    - (typical 1 AU conditions, $\rho_\mathrm{s}=10^3$ kg.m^{-3})
- Dust fluid w/ $\epsilon=\rho_\mathrm{d}/\rho_\mathrm{g} \ll 1$: advection-diffusion equation
  - $\dfrac{\partial\rho_\mathrm{d}}{\partial t} = D_\mathrm{t}\dfrac{\partial}{\partial z}\left[\rho_\mathrm{g}\dfrac{\partial}{\partial z}\left(\dfrac{\rho_\mathrm{d}}{\rho_\mathrm{g}}\right)\right] + \dfrac{\partial}{\partial z}(\Omega_\mathrm{K}\mathrm{St}\rho_\mathrm{d} z)$
  - steady-state solution if ρ_g=cst in dust layer
    - $\rho_\mathrm{d}(z) = \rho_\mathrm{d}(0)\mathrm{e}^{-z^2/2H_\mathrm{d}^2}, \ H_\mathrm{d} = \sqrt{\dfrac{\alpha}{\mathrm{St}+\alpha}}H$

Dubrulle+1995

Radial drift

- Small grains, St≪1
  - strong coupling, grains entrained by gas, $v_{\mathrm{d},\theta} \simeq v_{\mathrm{g},\theta} < v_{\mathrm{K}}$
  - centrifugal force insufficient to counter gravity
  - grains spiral inwards at terminal radial velocity
- Large particles, St≫1
  - weak perturbations, $v_{\mathrm{d},\theta} \simeq v_{\mathrm{K}} > v_{\mathrm{g},\theta}$
  - headwind ⇒ torque removes angular momentum
  - inwards drift on orbit with lower angular momentum $(j \propto \sqrt{r})$

Radial drift

- Equations of motion in disk mid-plane

$$\begin{cases} \dfrac{\mathrm{d}v_{\mathrm{d},r}}{\mathrm{d}t} - \dfrac{v_{\mathrm{d},\theta}^2}{r} & = & -\Omega_{\mathrm{K}}^2 r - \dfrac{(v_{\mathrm{d},r} - v_{\mathrm{g},r})}{t_{\mathrm{s}}} \\ \dfrac{1}{r}\dfrac{\mathrm{d}}{\mathrm{d}t}(r v_{\mathrm{d},\theta}) & = & -\dfrac{(v_{\mathrm{d},\theta} - v_{\mathrm{g},\theta})}{t_{\mathrm{s}}} \end{cases}$$

Radial drift

- Stationary solution in disk mid-plane
  - approximation: small $\eta = \frac{1}{2}\left(\frac{H}{r}\right)^2 \left(-\frac{\mathrm{d}\log P_\mathrm{g}}{\mathrm{d}\log r}\right) > 0$

$$\begin{cases} v_{\mathrm{g},r} &= v_\mathrm{visc} \\ v_{\mathrm{d},r} &= \underbrace{\frac{\mathrm{St}}{1+\mathrm{St}^2} v_\mathrm{drift}}_{\text{dominates for } \mathrm{St} \gg \alpha} + \underbrace{\frac{1}{1+\mathrm{St}^2} v_\mathrm{visc}}_{\text{dominates for } \mathrm{St} \ll \alpha} \end{cases}$$

$$\begin{cases} v_\mathrm{drift} &= -2\eta\, v_\mathrm{K} < 0 \\ v_\mathrm{visc} &= \dfrac{\dfrac{\partial}{\partial r}\left(\rho_\mathrm{g} \nu r^3 \dfrac{\partial \Omega_\mathrm{K}}{\partial r}\right)}{r\rho_\mathrm{g} \dfrac{\partial}{\partial r}\left(r^2 \Omega_\mathrm{K}\right)} < 0 \end{cases}$$

Nakagawa+1986

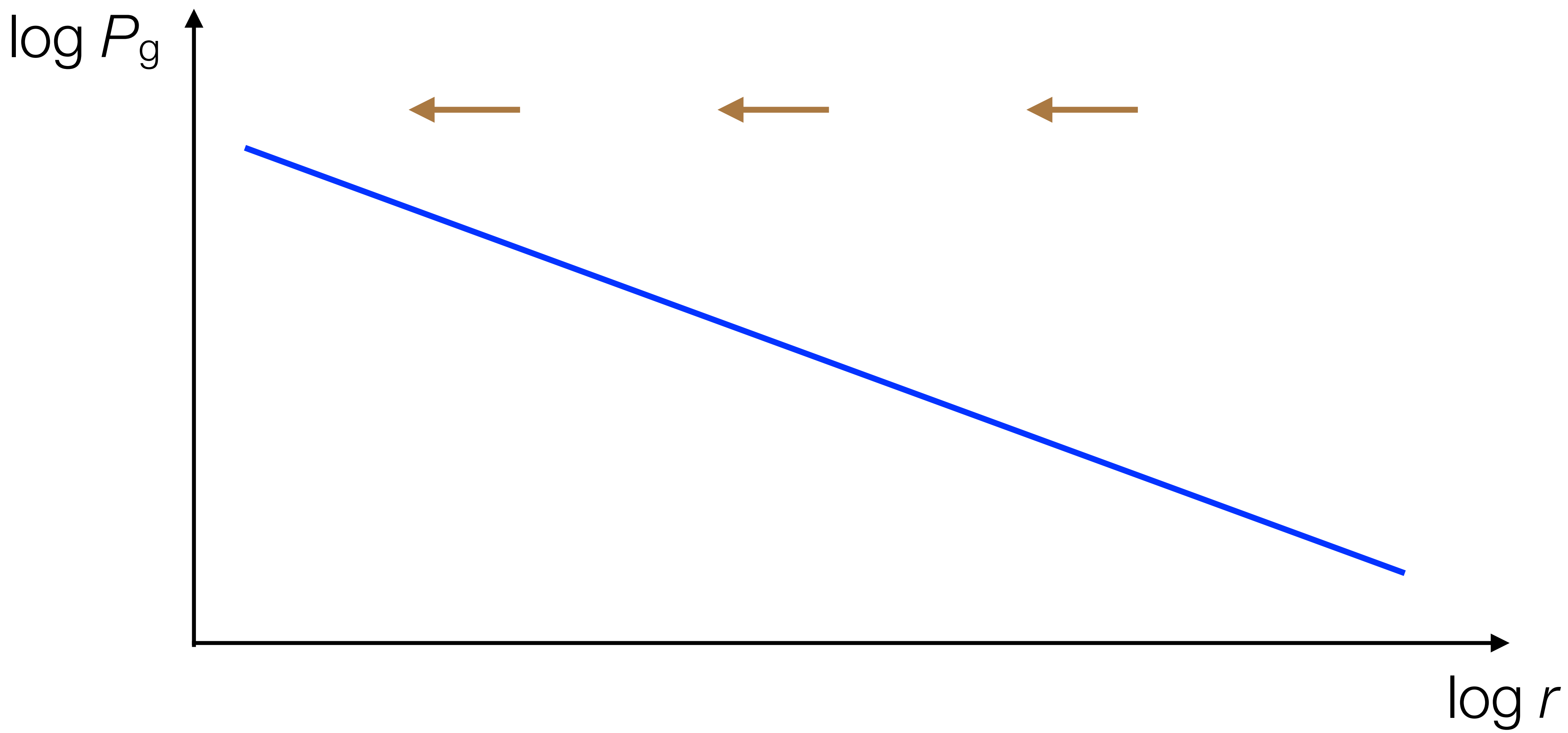
$$v_\mathrm{drift} \sim \frac{1}{\alpha} v_\mathrm{visc}$$

Lynden-Bell+Pringle1974

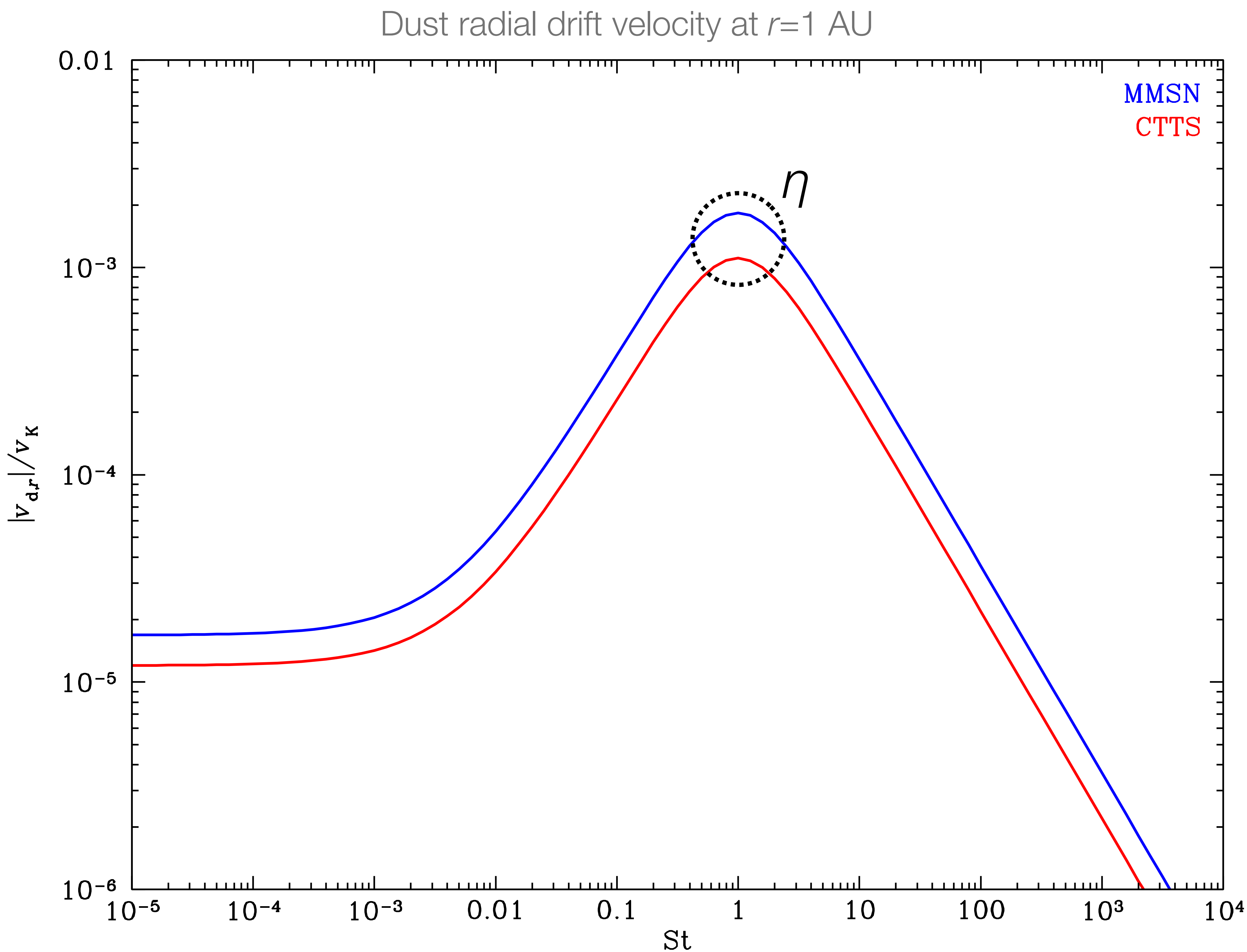
Radial drift

- Dust radial drift velocity
  - $\mathrm{St} \gg \alpha \Rightarrow v_{\mathrm{d},r} = \frac{\mathrm{St}}{1+\mathrm{St}^2}\left(\frac{H}{r}\right)^2 \frac{\mathrm{d}\log P_\mathrm{g}}{\mathrm{d}\log r} v_\mathrm{K}$

  ⇒ grains drift towards the pressure maximum



Radial drift



The radial-drift barrier

- Drift time-scale: $t_{\rm drift} = \dfrac{r}{v_{{\rm d},r}} = \dfrac{1+{\rm St}^2}{\eta {\rm St}} \dfrac{t_{\rm K}}{2}$
  - $T_{\rm drift} = \dfrac{t_{\rm drift}}{t_{\rm K}} \sim \dfrac{1+{\rm St}^2}{\eta {\rm St}} \sim \dfrac{T_{\rm set}}{\eta}$
  - thin disks: $\eta \ll 1 \Rightarrow T_{\rm drift} \gg T_{\rm set}$
- Minimum drift time-scale for St~1: $t_{\rm drift}^{\rm min} = \dfrac{t_{\rm K}}{\eta}$
  - MMSN@1 AU: $t_{\rm drift}^{\rm min} \sim 100\ {\rm yr}$ for $s({\rm St}=1) \sim 1\ {\rm m}$

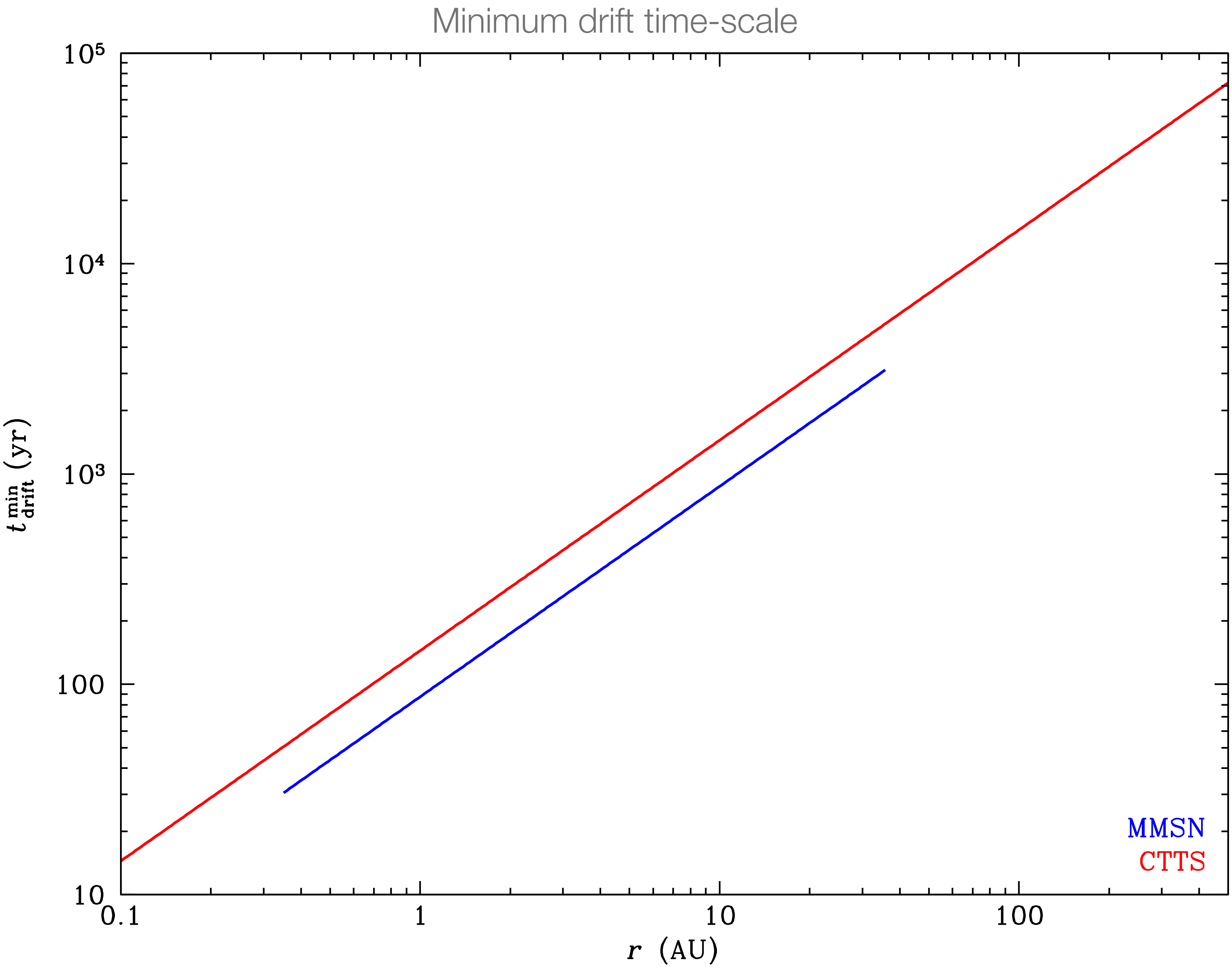
    ⇒ "meter-size" barrier

Weidenschilling 1977

  - but St depends on disk conditions

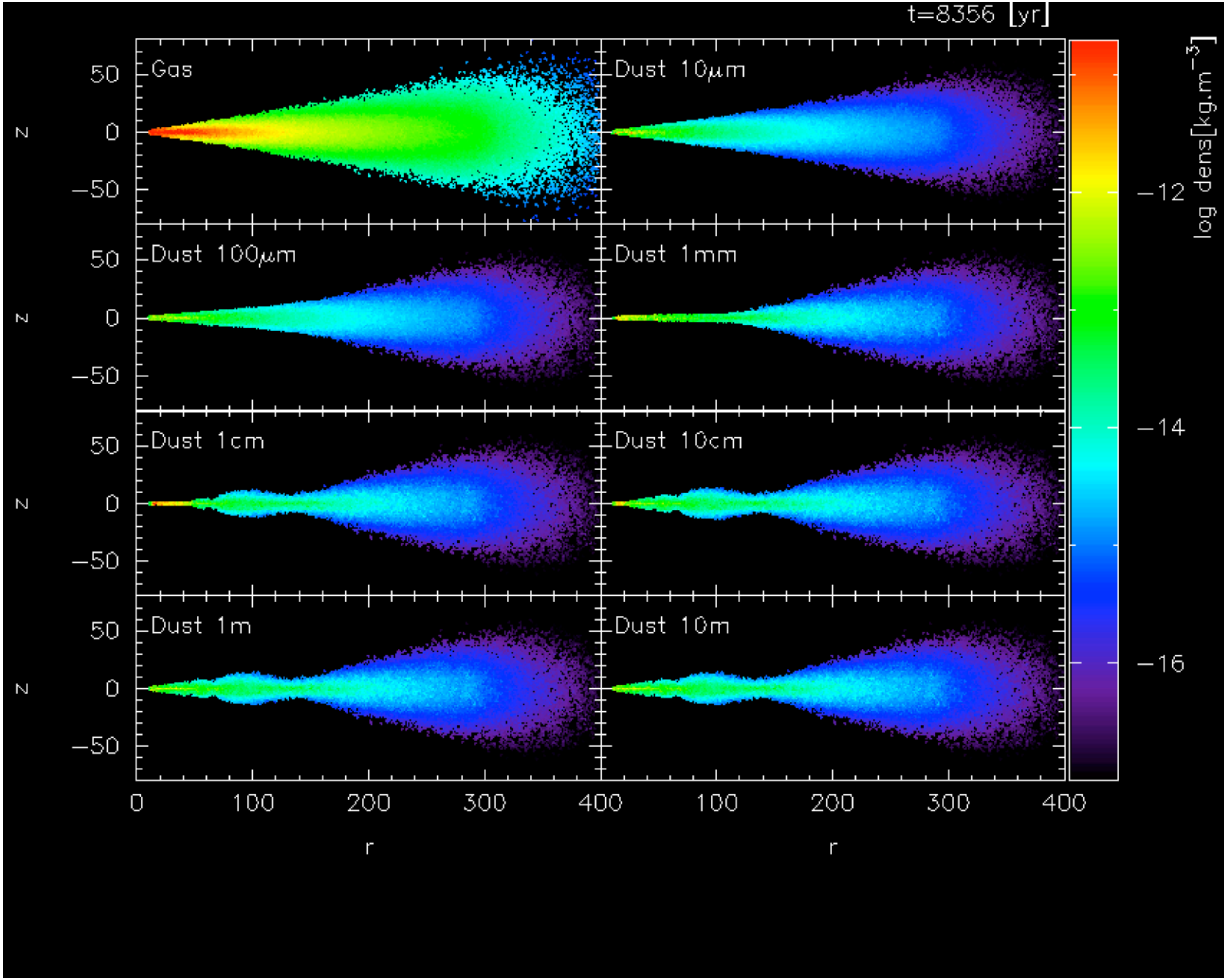
    ⇒ radial-drift barrier

The radial-drift barrier



Radial drift and vertical settling

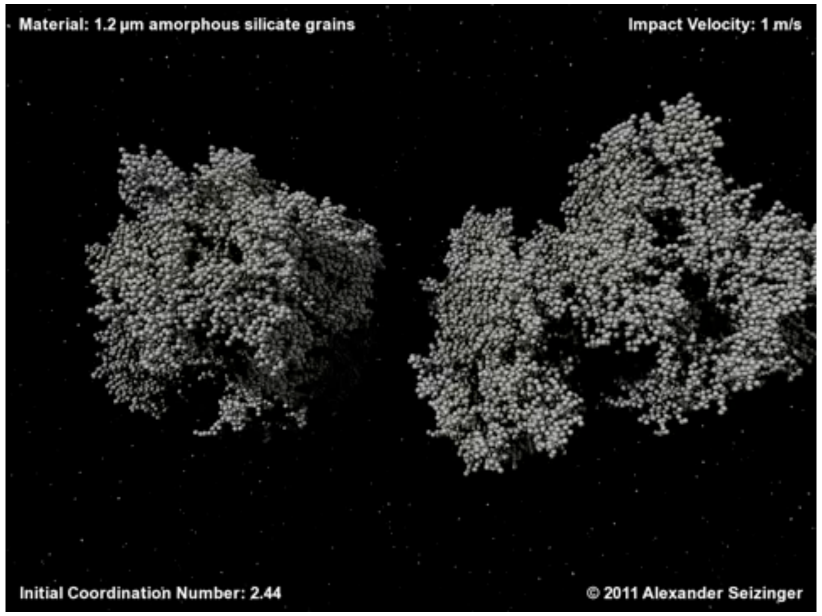
$M_{\rm disk} = 0.01\ M_\odot,\ r_{\rm out} = 400\ {\rm AU},\ \epsilon_0 = 1\%,\ \rho_{\rm s} = 1000\ {\rm kg\,m^{-3}}$, initial state: p=3/2, q=3/4



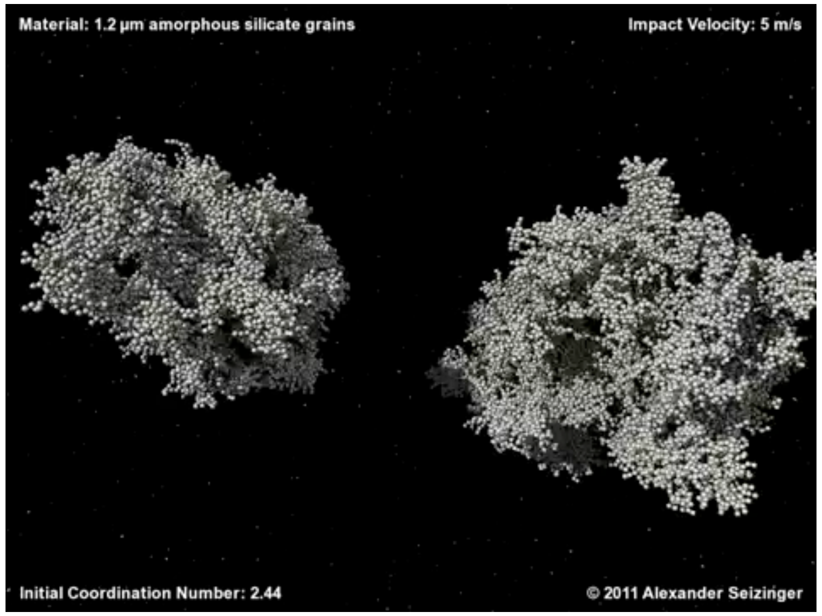
Barrière-Fouchet+2005

The fragmentation and bouncing barriers

- Material properties ⇒ fragmentation threshold v_{frag}
  - Growth when $v_{\mathrm{rel}} < v_{\mathrm{frag}}$



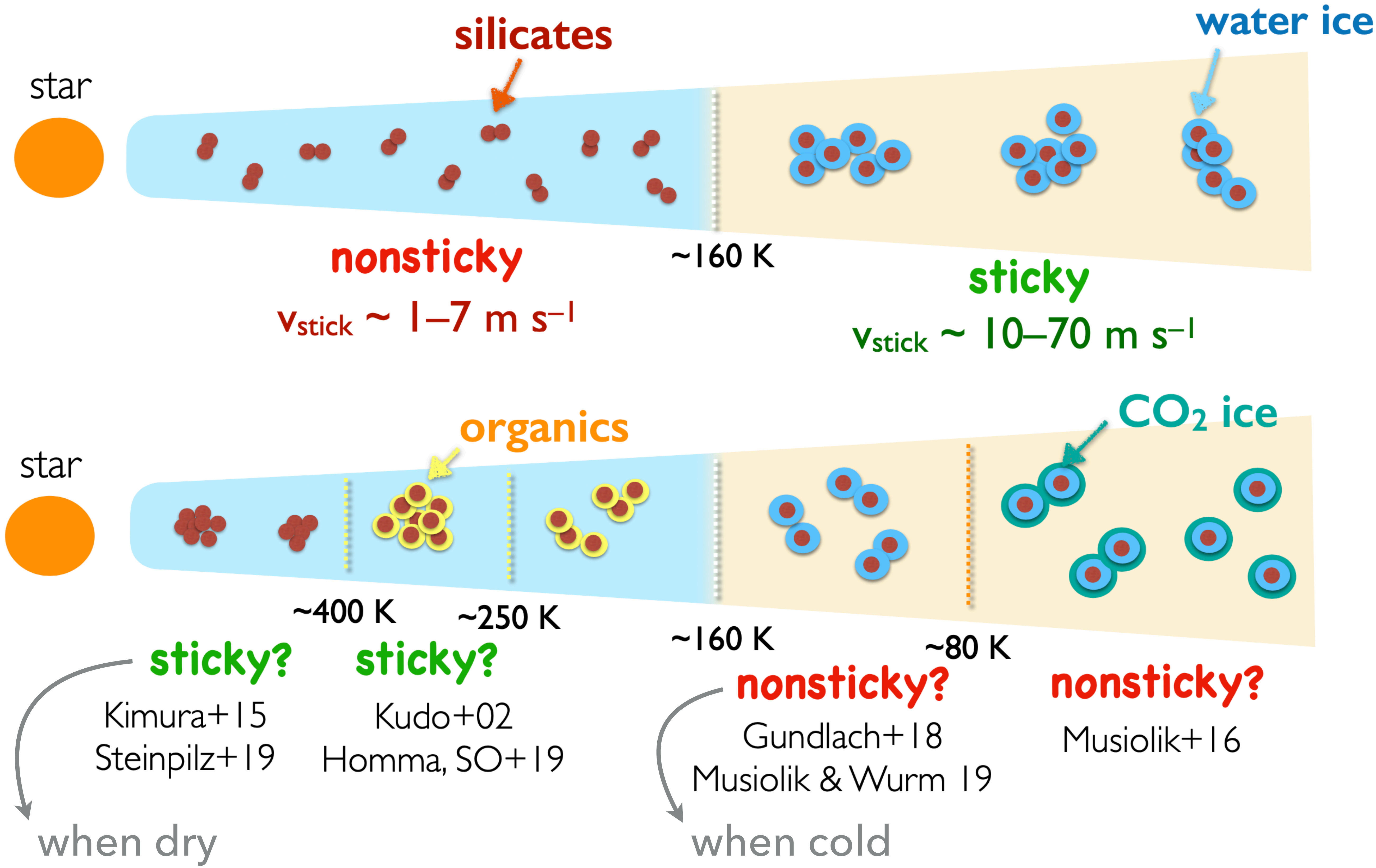
  - Fragmentation when $v_{\mathrm{rel}} > v_{\mathrm{frag}}$



  - Bouncing when $v_{\mathrm{rel}} \lesssim v_{\mathrm{frag}}$

Setzinger 2011

The fragmentation and bouncing barriers



Okuzumi 2020

Growth and fragmentation

- Coagulation equation: Smoluchowski (1916)

$$\Rightarrow \frac{\partial n(m)}{\partial t} = \frac{1}{2}\int_0^m K(m', m-m')n(m')n(m-m')\mathrm{d}m' - n(m)\int_0^{+\infty} K(m', m)n(m')\mathrm{d}m'$$

  - physics encoded in reaction kernel *K*, for coagulation:

$$K(m_1, m_2) = P(m_1, m_2, v_{\rm rel})v_{\rm rel}(m_1, m_2)\sigma(m_1, m_2)$$

  - integro-differential equation difficult to solve

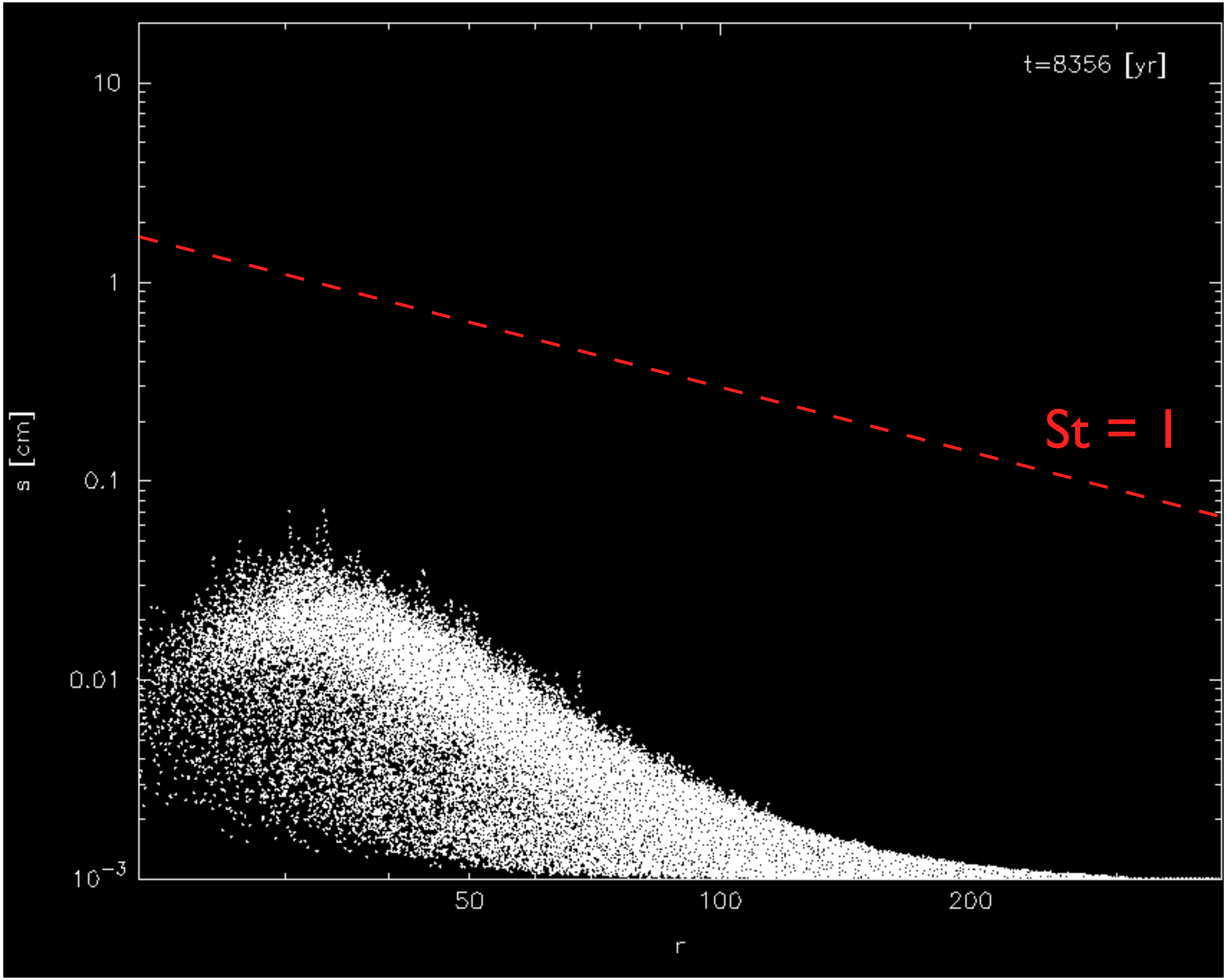
Locally mono-disperse size distribution: growth

- Perfect sticking of equal-mass compact grains
  - $\frac{\mathrm{d}m_\mathrm{d}}{\mathrm{d}t} = \frac{m_\mathrm{d}}{t_\mathrm{coll}} = 4\pi s^2 \epsilon \rho_\mathrm{g} v_\mathrm{rel} \Rightarrow \frac{\mathrm{d}s}{\mathrm{d}t} = \epsilon \frac{\rho_\mathrm{g}}{\rho_\mathrm{s}} v_\mathrm{rel}$
- Turbulent coupling with gas
  - $v_\mathrm{rel} = f(\alpha, \mathrm{St}) c_\mathrm{s}$
  - Prandtl-like turbulence: $v_\mathrm{rel} = \sqrt{\alpha} f'(\mathrm{St}) c_\mathrm{s}$
  - Turbulence as a correlated noise:
    - $v_\mathrm{rel} \propto \sqrt{\alpha} \frac{\sqrt{\mathrm{St}}}{1 + \mathrm{St}} c_\mathrm{s}$

Stepinski & Valageas 1997

Locally mono-disperse size distribution: growth

$M_{\rm disk} = 0.01\ M_{\odot},\ r_{\rm out} = 400\ {\rm AU},\ \epsilon_0 = 1\%,\ \rho_{\rm s} = 1000\ {\rm kg\,m^{-3}}$, initial state: *p*=3/2, *q*=3/4



Laibe+2008

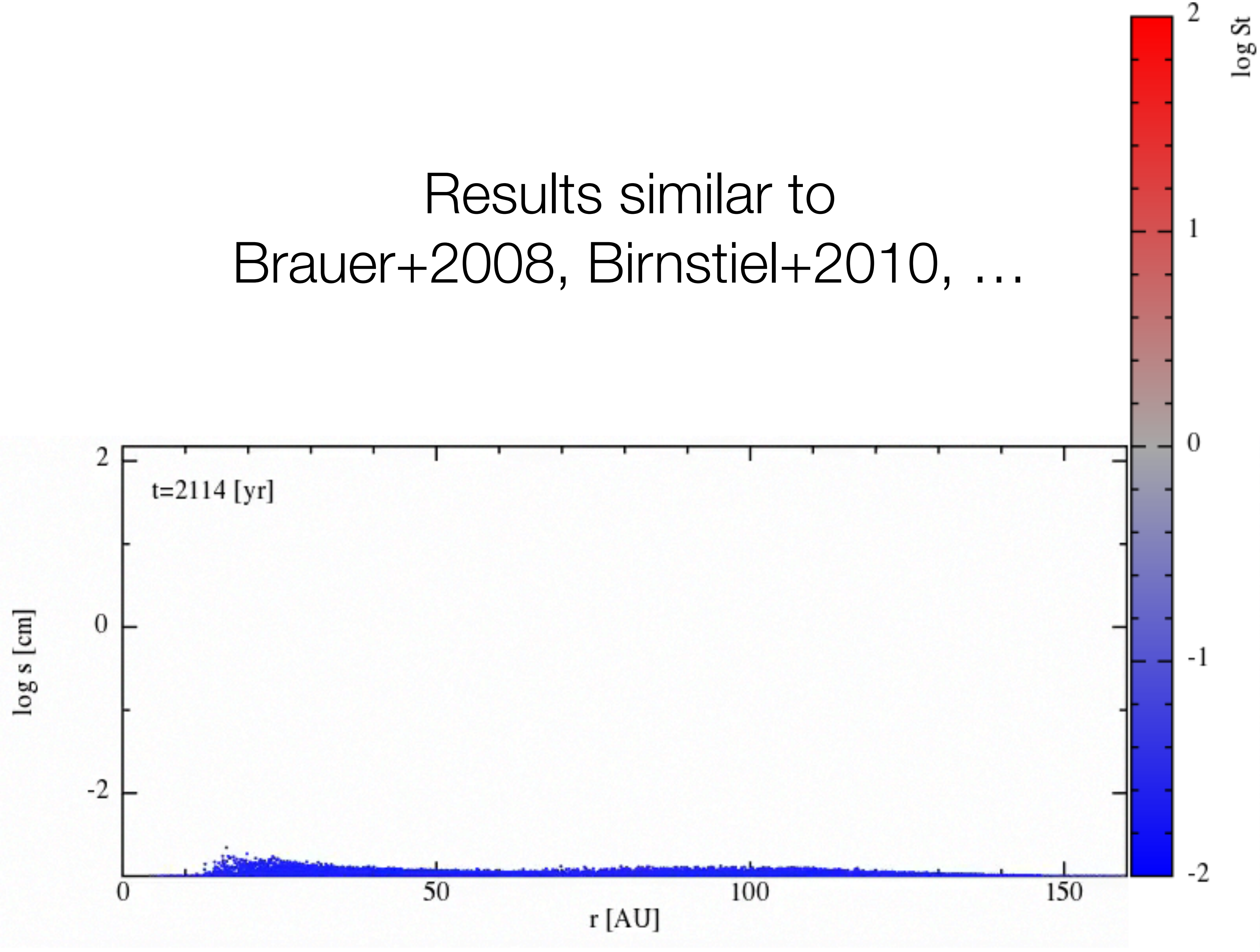
Growth and fragmentation

- Fragmentation to very small sizes for $v_{\rm rel} > v_{\rm frag}$
  - $\dfrac{{\rm d}m_{\rm d}}{{\rm d}t} = -\dfrac{m_{\rm d}}{t_{\rm coll}} \Rightarrow \dfrac{{\rm d}s}{{\rm d}t} = -\epsilon\dfrac{\rho_{\rm g}}{\rho_{\rm s}}v_{\rm rel}$
- Disk mid-plane: ${\rm St}(z=0) = \dfrac{\pi\rho_{\rm s}s}{2\Sigma_{\rm g}}$
- Simple interpretation for *p*=0
  - ${\rm St}(z=0) \propto s$
  - $v_{\rm rel} \propto c_{\rm s} \propto r^{-1/2}$
    - inner disk: $v_{\rm rel} > v_{\rm frag} \Rightarrow$ fragmentation
    - outer disk: $v_{\rm rel} < v_{\rm frag} \Rightarrow$ growth

Growth and fragmentation

$M_{\rm disk} = 0.01\ M_\odot,\ r_{\rm out} = 160\ \mathrm{AU},\ p = 0,\ q = 1,\ \epsilon_0 = 1\%,\ \rho_{\rm s} = 1000\ \mathrm{kg\,m^{-3}},\ v_{\rm frag} = 15\ \mathrm{m\,s^{-1}}$

Results similar to
Brauer+2008, Birnstiel+2010, …



Gonzalez+2017

Planetesimal formation via coagulation

- Growth to mm-cm size easy
  - size distribution set by growth/fragmentation balance

- Typical v_{rel} including differential radial drift and turbulence
  Weidenschilling+Cuzzi1993
  - bouncing or fragmentation rather than growth

  ⇒ growth beyond cm difficult

- Turbulent disk: distribution of v_{rel}
  - low-velocity tail ⇒ net growth to cm- or m-size
    Garaud+2013
  - larger objects grow via sweep-up of mm grains
    Windmark+2012

- Fragmentation needed to maintain small grains: observed

Planetesimal formation via coagulation

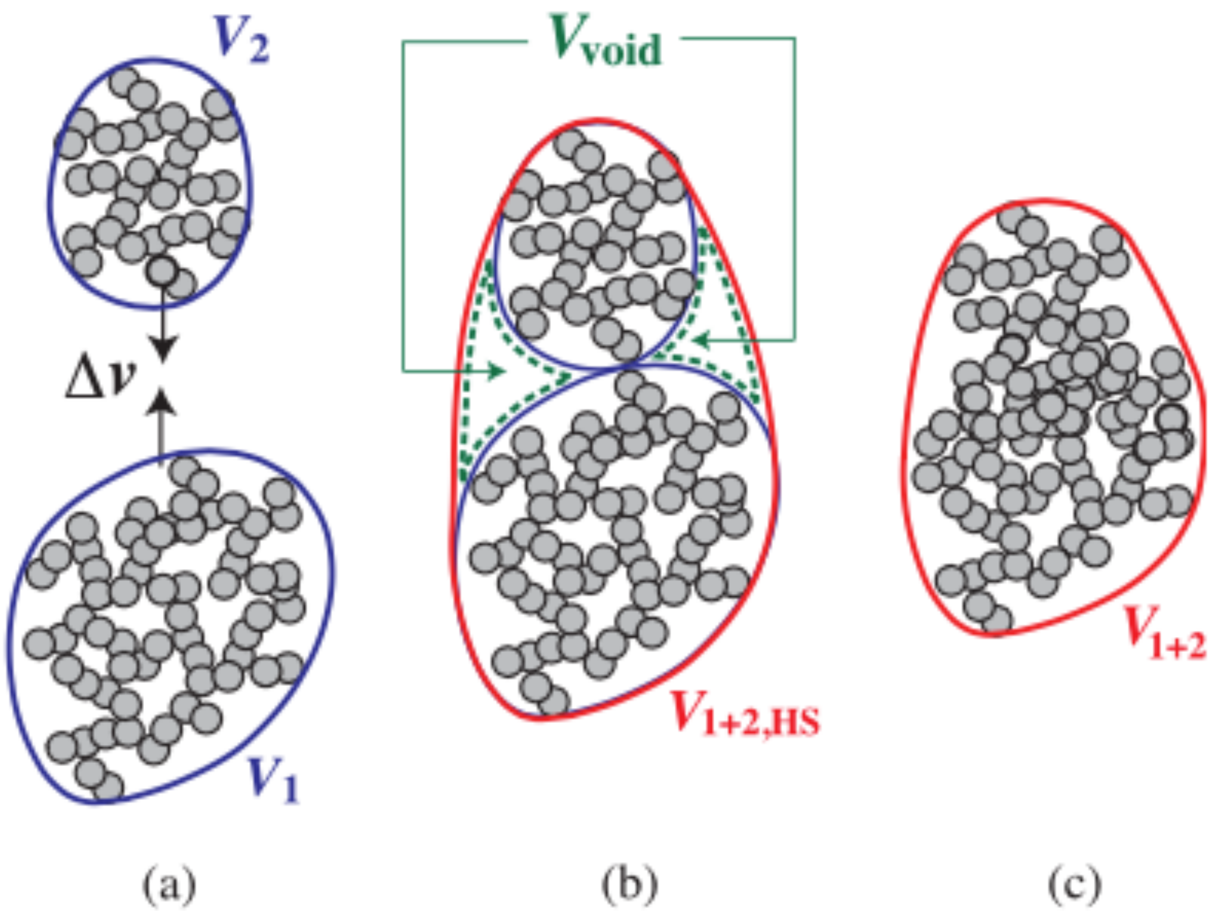
- Possible solutions
  - accelerate grain growth
    - grain porosity
  - slow down or stop radial drift
    - dust pile-up in particle traps

Grain porosity

Collisional evolution

Filling factor:

$$\phi = \frac{V_{\rm mat}}{V} = \frac{\rho}{\rho_{\rm s}}$$



Okuzumi+2012

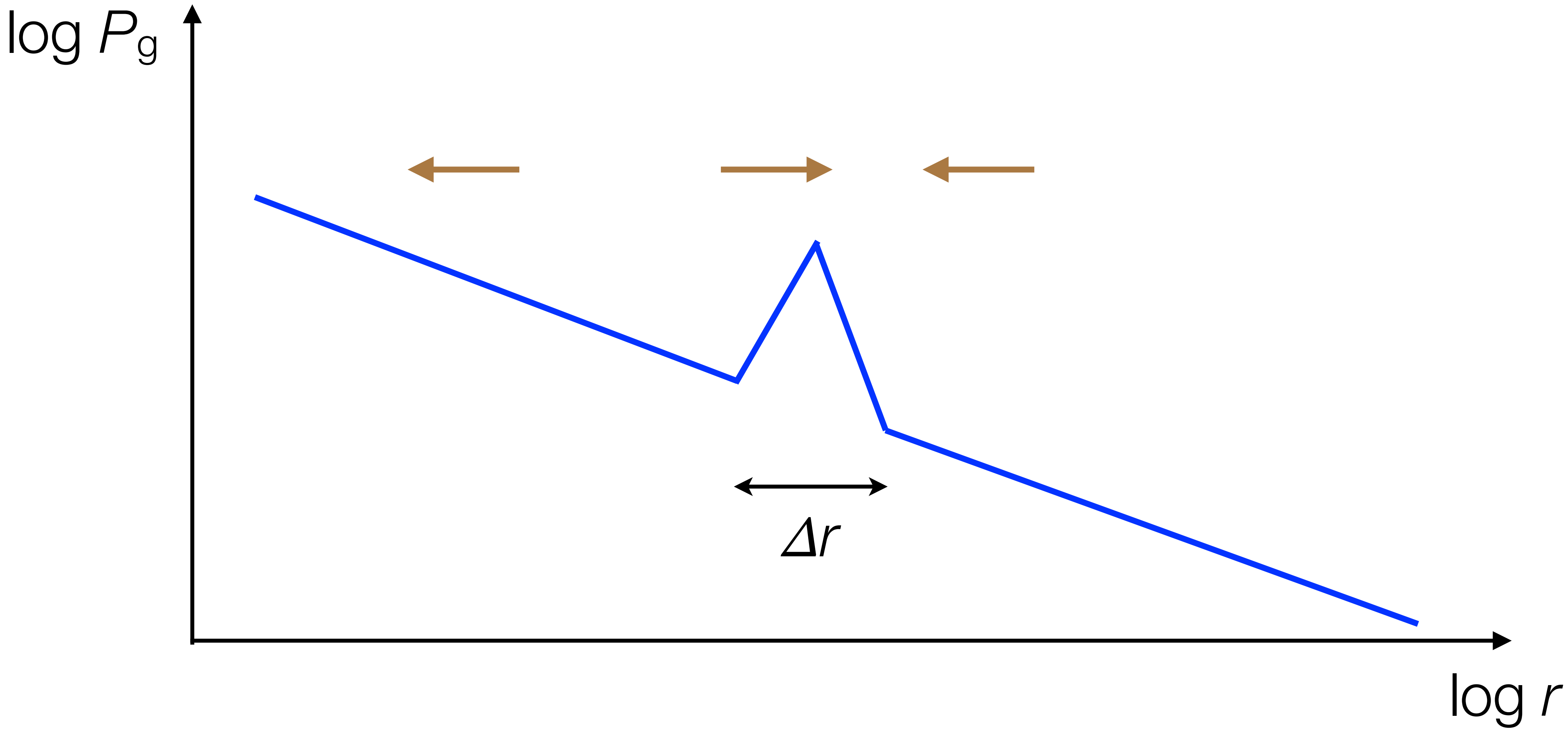
Porous grains are larger ⇒ faster growth

$$\mathrm{St} = \frac{\Omega_{\rm K}\rho_{\rm s}\phi s}{\rho_{\rm g} v_{\rm K}} \Rightarrow \text{degeneracy}$$

Particle traps

- If pressure maximum exists over scale Δr

  $\Rightarrow$ grains drift towards the pressure maximum



- local pressure gradient $\sim P_g/\Delta r$ > global gradient $\sim P_g/r$

  $\Rightarrow$ pile-up timescale < global drift time by a factor $(r/\Delta r)^2$

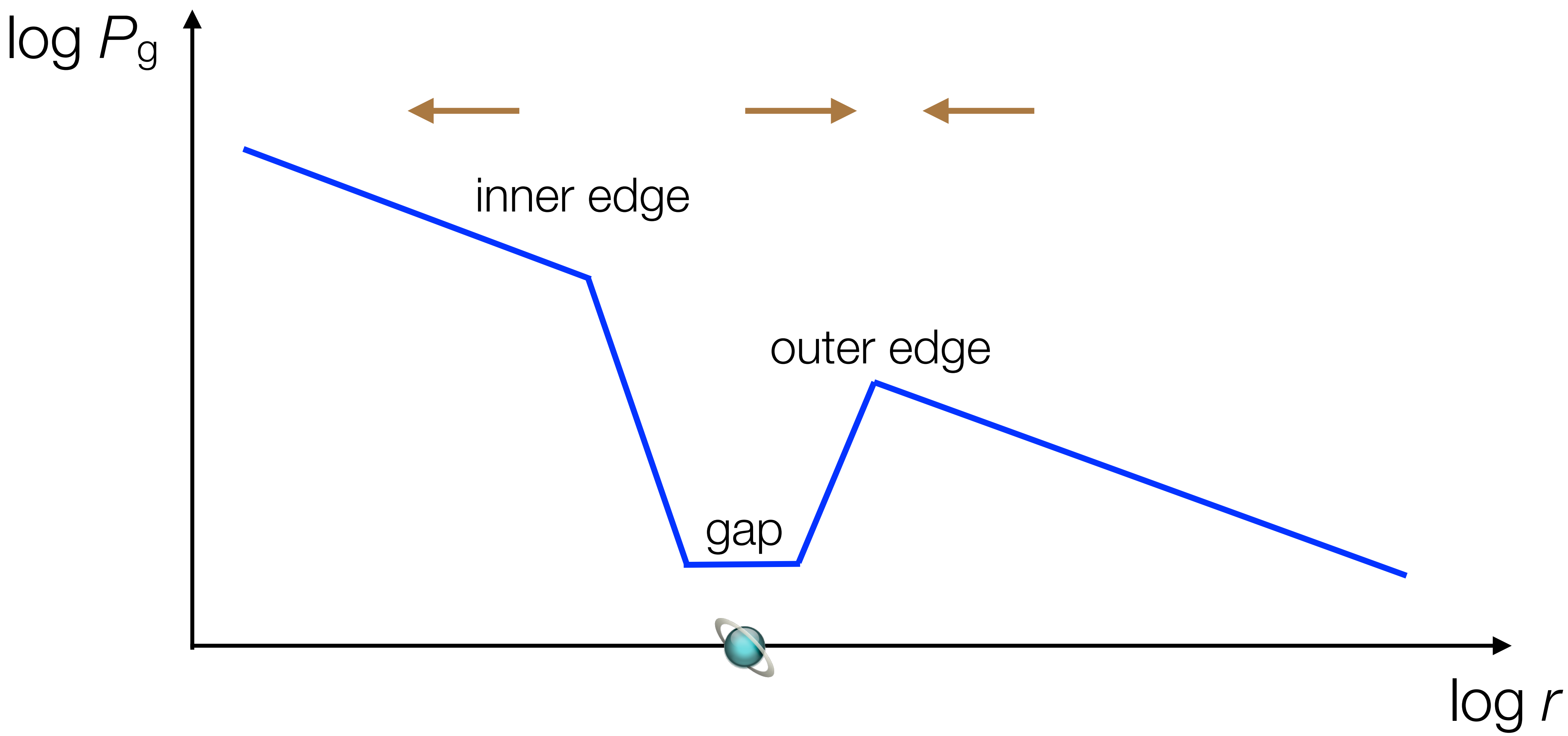
Particle traps

- Pressure maxima in the disk
  - vortices
    Barge & Sommeria 1995, Regály+2012, Méheut+2013
  - snow line
    Kretke & Lin 2007
  - dead zone inner edge
    Dzyurkevich+2010
  - planet gap outer edge
    de Val-Borro+2007, Fouchet+2007,2010, Gonzalez+2012, Zhu 2012,2014
  - zonal flows in MRI turbulence
    Bethune+2016
  - "bumpy" gas surface density
    Pinilla+2012

⇒ Dust concentrations

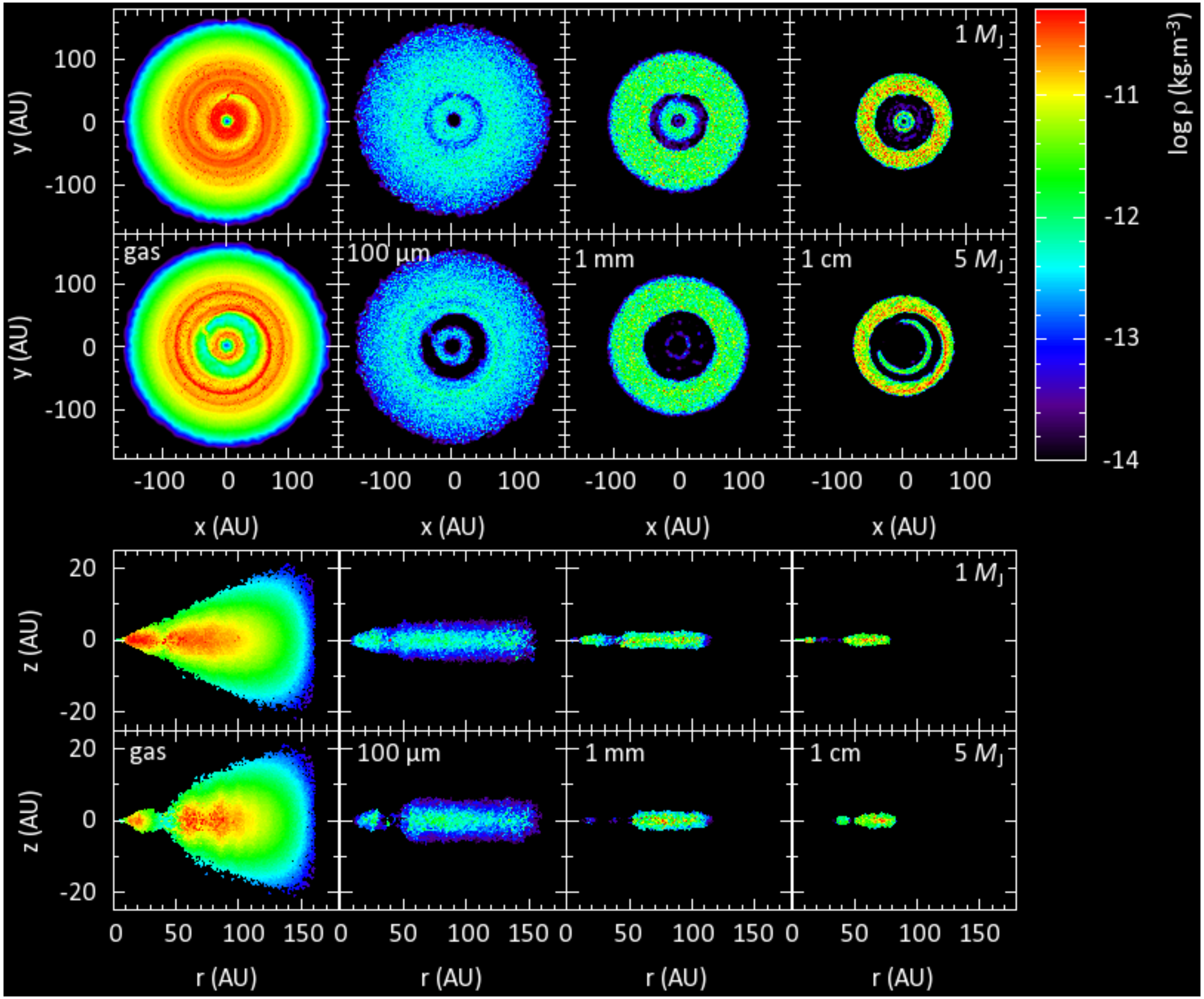
- ϵ increases
- v_{rel} decreases

Planet gaps



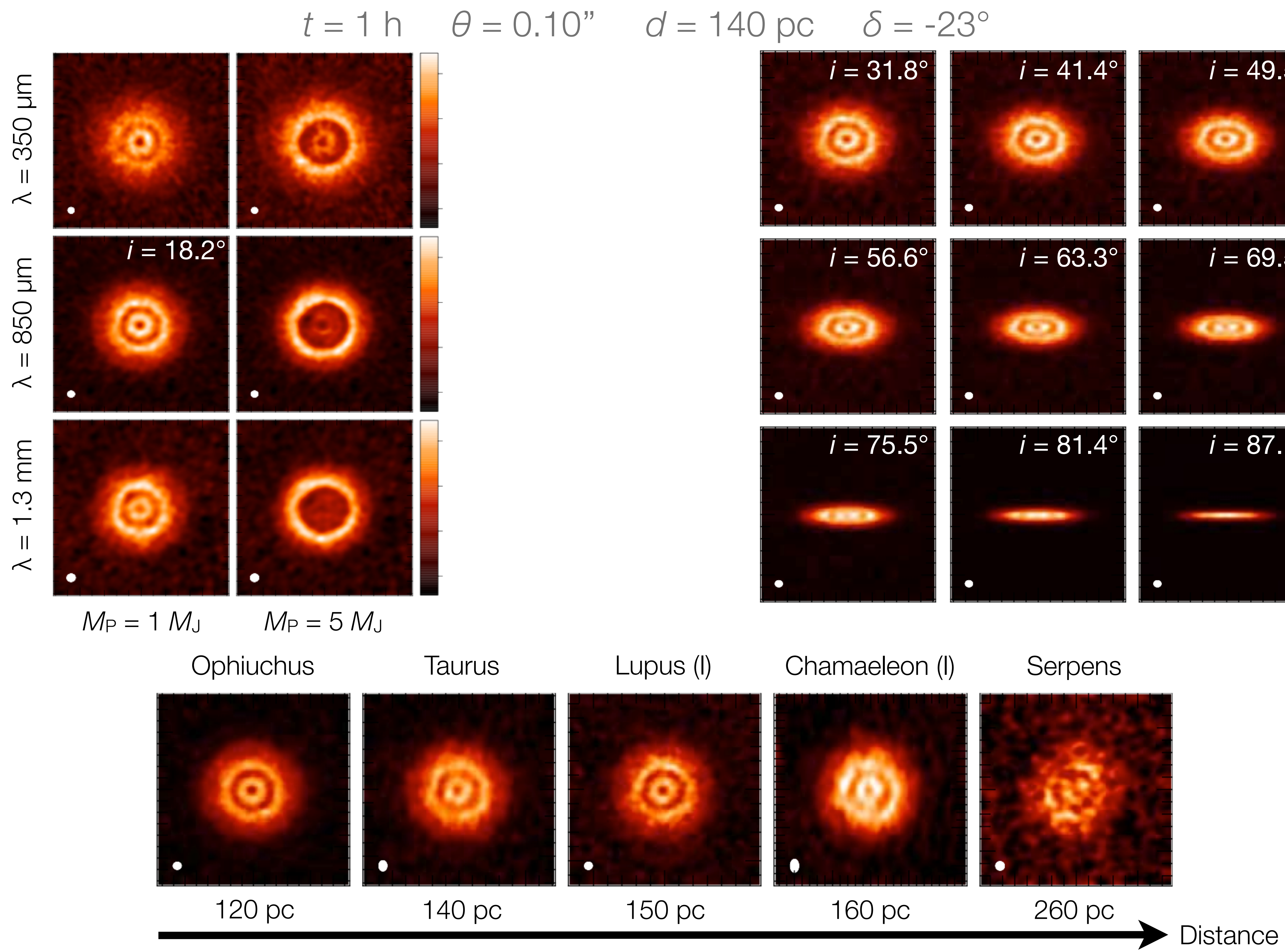
Disk with planet: grains of constant size

$$M_{\rm disk} = 0.01\ M_\odot,\ r_{\rm out} = 160\ {\rm AU},\ p = 0,\ q = 1,\ \epsilon_0 = 1\%,\ \rho_{\rm s} = 1000\ {\rm kg\,m^{-3}}$$
$$M_{\rm P} = 1\ \&\ 5\ M_{\rm J},\ a = 40\ {\rm AU},\ s = 100\ \mu{\rm m},\ 1\ {\rm mm},\ 1\ {\rm cm}$$



Fouchet+2010

Disk with planet: ALMA simulated images

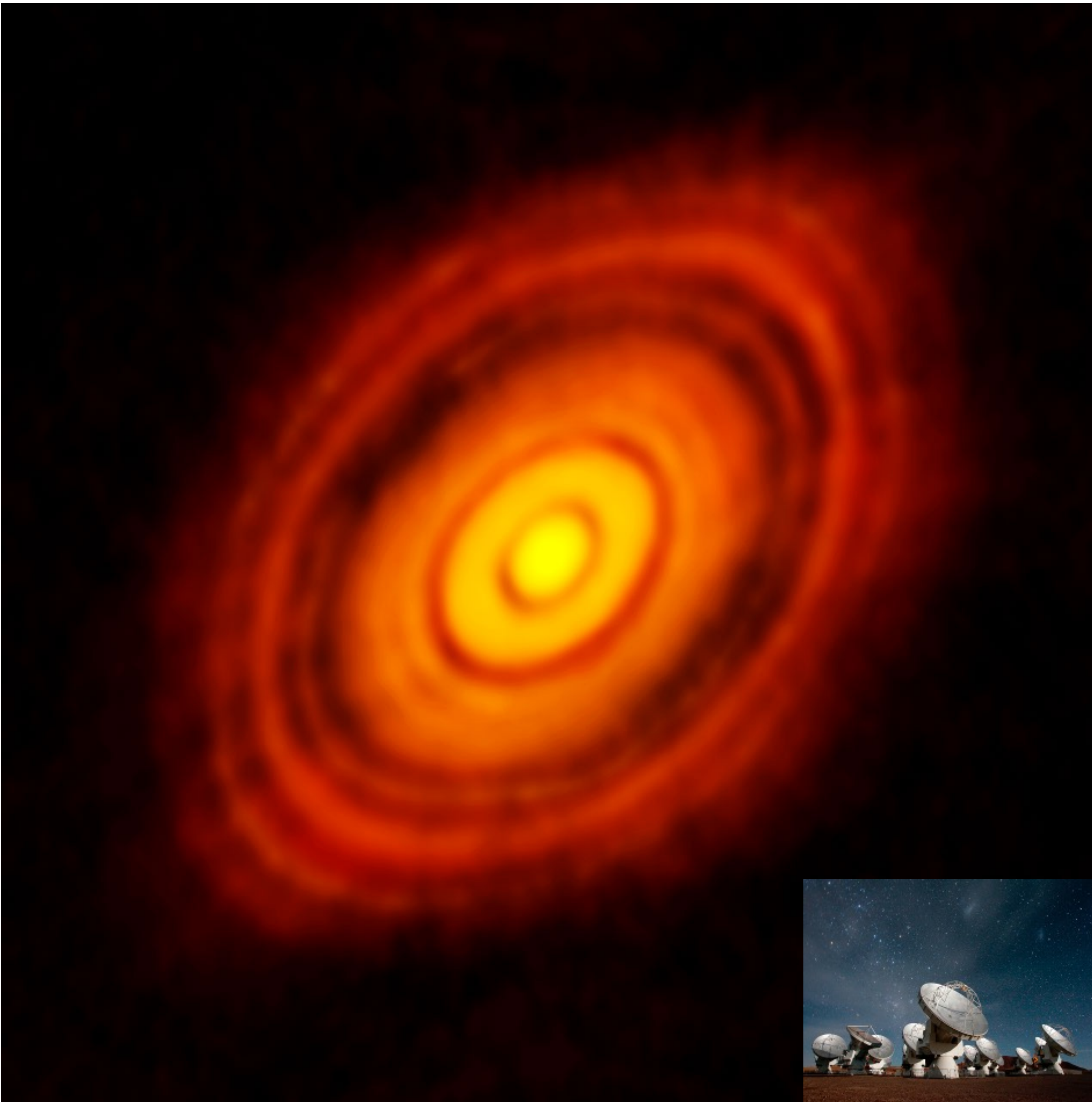


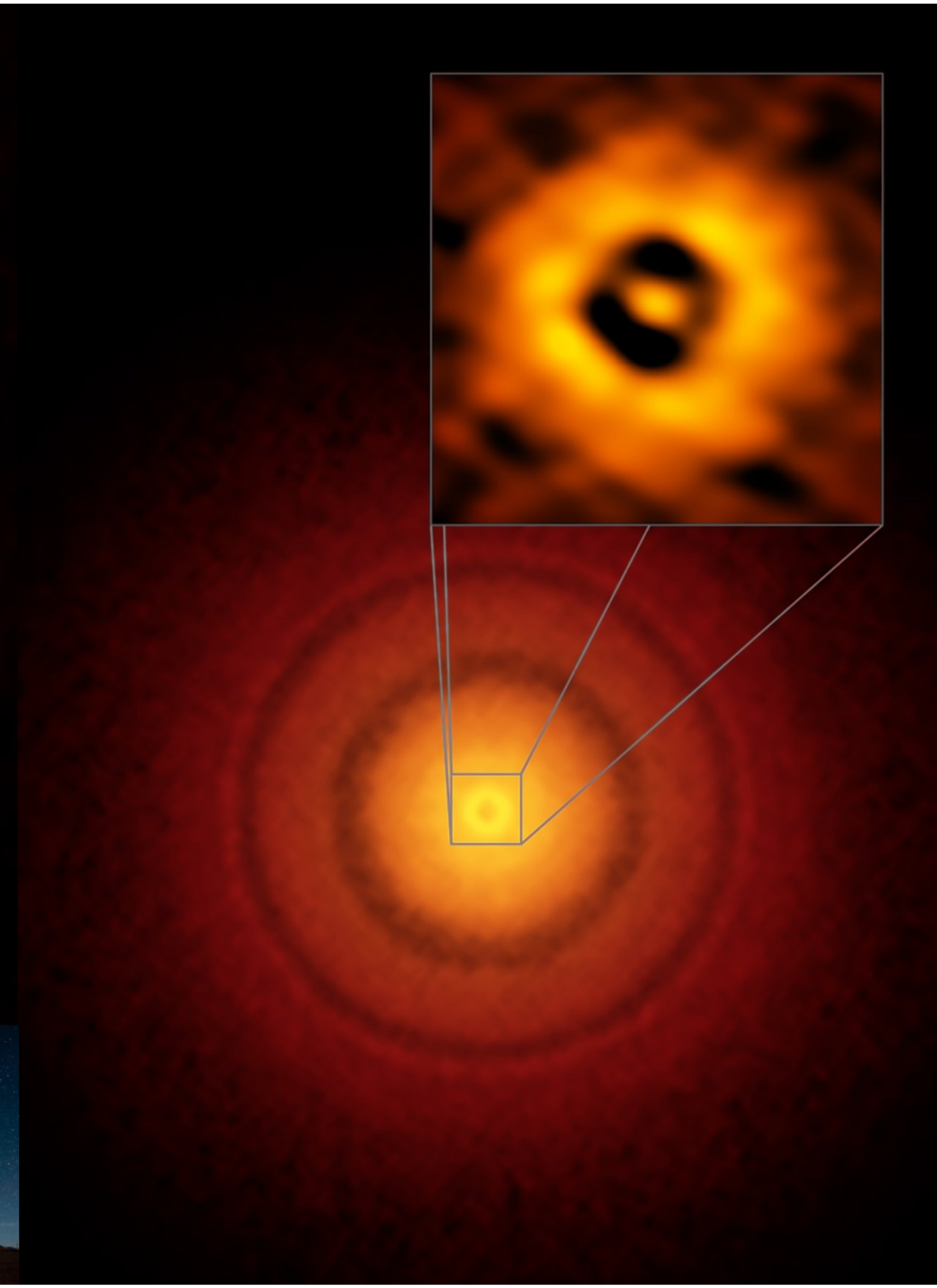
Gonzalez+2012

Planet gaps

HL Tau

TW Hya





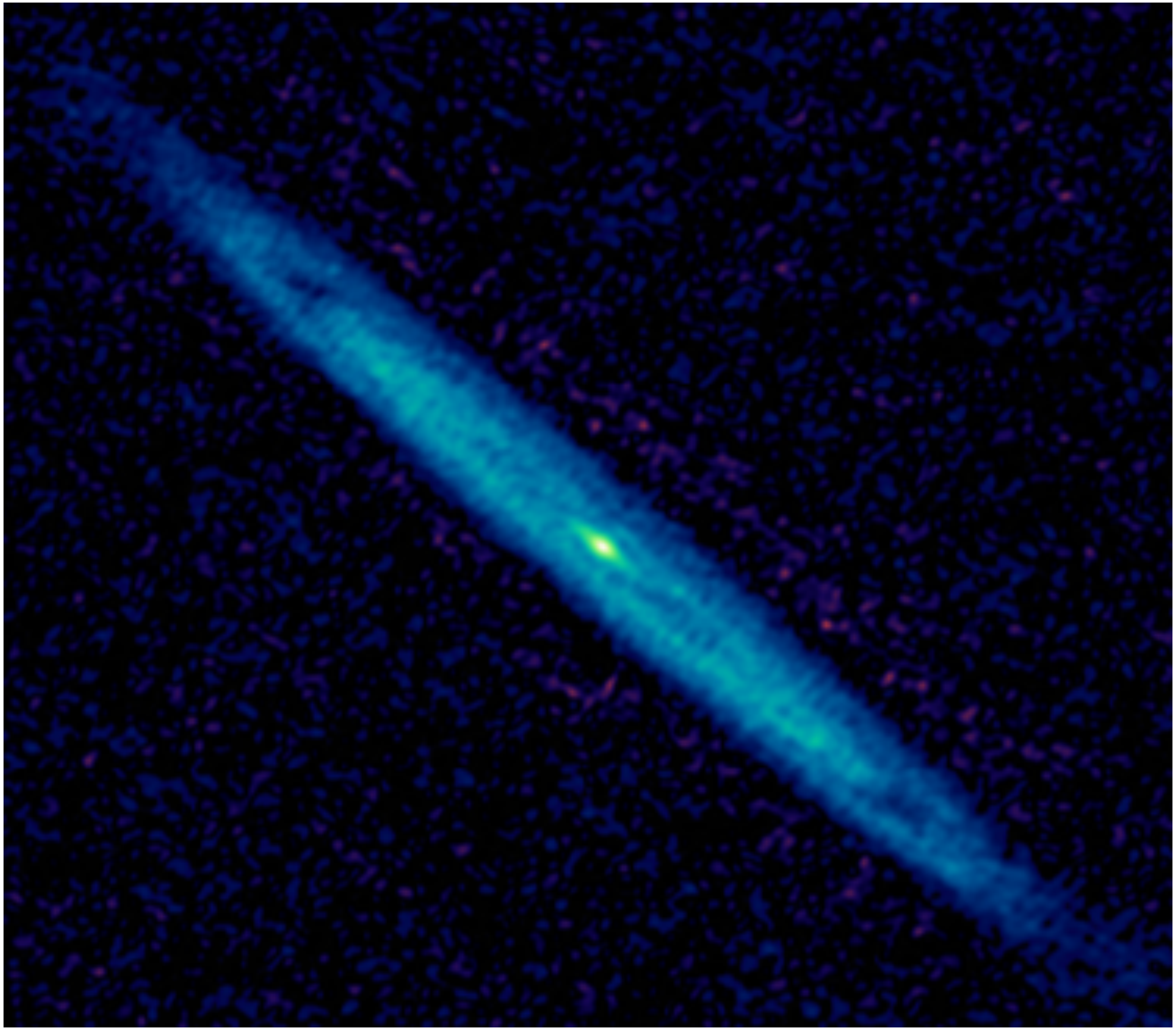
ALMA Partnership+2015

ALMA

Andrews+2016

Planet gaps

Oph 163131



ALMA

Villenave+2022